Find the recurrence relation for the series solutions of the DE $y'' + 2xy' + 2y = 0$ about $x_0 = 0$.

$$y = \sum_{n=0}^{\infty} c_n x^n \quad , \quad y' = \sum_{n=1}^{\infty} n c_n x^{n-1} \quad , \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

Substituting in the DE,

$$y'' + 2xy' + 2y =$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + 2x \sum_{n=1}^{\infty} n c_n x^{n-1} + 2 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\underbrace{\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}}_{\text{Put } k=n-2} + \underbrace{\sum_{n=1}^{\infty} 2n c_n x^n}_{k=n} + \underbrace{\sum_{n=0}^{\infty} 2 c_n x^n}_{k=n} = 0$$

$$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k + \sum_{k=1}^{\infty} 2k c_k x^k + \sum_{k=0}^{\infty} 2 c_k x^k = 0$$

$$2c_2 + \sum_{k=1}^{\infty} (k+2)(k+1) c_{k+2} x^k + \qquad + 2c_0 + \sum_{k=1}^{\infty} 2 c_k x^k = 0$$

$$2c_2 + 2c_0 + \sum_{k=1}^{\infty} \left[ (k+2)(k+1) c_{k+2} + 2k c_k + 2c_k \right] x^k = 0$$

$$\therefore 2c_2 + 2c_0 = 0 \quad \& \quad (k+2)(k+1) c_{k+2} + 2(k+1) c_k = 0$$

$$\Rightarrow c_2 = -c_0 \quad \& \quad c_{k+2} = -\frac{2}{k+2} c_k \quad , \quad k = 1, 2, 3, \ldots$$