

Math 202 Quiz # 4

Name: Solution Sr. # _____ Section # _____1. Determine whether the following set of functions is linearly independent on $(-\infty, \infty)$.

$$f_1(x) = 4x - 3x^2, \quad f_2(x) = x^2, \quad f_3(x) = x.$$

Method 1

$$\text{Consider } af_1(x) + bf_2(x) + cf_3(x) = 0$$

$$a(4x - 3x^2) + bx^2 + cx = 0$$

We can find constants a, b, c , not all zero, satisfying the above eq.

$$\text{i.e. } a = 1, \quad b = 3, \quad c = -4$$

$$1 \cdot f_1(x) + 3f_2(x) - 4f_3(x) = 0$$

$\therefore \{f_1, f_2, f_3\}$ is linearly dep.

Method 2

You can easily see that $f_1(x) = 4f_3(x) - 3f_2(x)$

i.e. one function is a linear combination of the others

$\Rightarrow \{f_1, f_2, f_3\}$ is linearly dep

Method 3

$$\text{Calculate } W(f_1, f_2, f_3) = \begin{vmatrix} 4x - 3x^2 & x^2 & x \\ 4 - 6x & 2x & 1 \\ -6 & 2 & 0 \end{vmatrix} \stackrel{\text{---}}{=} \stackrel{\text{---}}{=} 0$$

$\Rightarrow \{f_1, f_2, f_3\}$ is linearly dep.