Math 202 Quiz # 3

Name: Solution Serial # _____ Section # ____

A small metal bar, whose initial temperature was 20° C, is dropped into a large container of boiling water. How long will it take the bar to reach 90° C if it is known that its temperature increases 2° in one second?

$T(0) = 20$
$T_m = 100$
$T(1) = 22$
at $T(t) = 90$
$t = ?$

Recall: $\frac{dT}{dt} = K(T - T_m) \Rightarrow T = T_m + Ce^{Kt}$

$\therefore T(t) = 100 + Ce^{Kt}$

$T(0) = 100 + C = 20 \Rightarrow \boxed{C = -80}$

$\therefore T(t) = 100 - 80e^{Kt}$

$T(1) = 100 - 80e^{K} = 22$

$\Rightarrow 80e^{K} = 78 \Rightarrow e^{K} = \frac{78}{80} = \frac{39}{40} \Rightarrow \boxed{K = \ln\frac{39}{40}}$

$\therefore T(t) = 100 - 80e^{t(\ln\frac{39}{40})}$

To find t when $T(t) = 90$, we solve

$100 - 80e^{t(\ln\frac{39}{40})} = 90$

$80e^{t(\ln\frac{39}{40})} = 10 \Rightarrow t\ln\frac{39}{40} = \ln\frac{1}{8} = -\ln 8$

$t = \frac{-\ln 8}{\ln\frac{39}{40}} \simeq 82.1$ seconds.