Math 202 Quiz # 2

Name: Solution I.D. # ______ Section # ____

1. Determine, without solving, whether the following DE possesses a unique solution through the point (2,-3): $\frac{dy}{dx} = \sqrt{y^2 - 9}$

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$$\frac{dy}{dx} = \sqrt{y^2-9} = f(x)$$

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{y}{\sqrt{y^2-9}}$$

$\because \frac{\partial f}{\partial y}$ is not continuous when $y=-3$

i.e. $\frac{\partial f}{\partial y}$ is not continuous at $(2,-3) \Rightarrow$ the DE doesnot possess a unique solution through $(2,-3)$.

2. Solve the initial value problem: $\frac{dx}{dt} = 4(x^2+1),\ x(\pi/4) = 1$

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$$\frac{dx}{x^2+1} = 4\,dt$$

$$\tan^{-1} x = 4t + C$$

using the condition $x(\frac{\pi}{4}) = 1 \Rightarrow \tan^{-1} 1 = 4(\frac{\pi}{4}) + C$

$$\Rightarrow C = -\frac{3\pi}{4}$$

$\therefore$ the solution is $\tan^{-1} x = 4t - \frac{3\pi}{4}$.

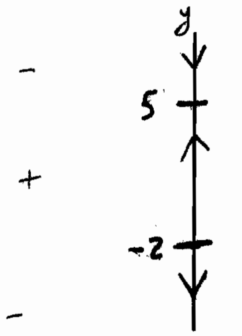
2. Consider the autonomous first order differential equation $\frac{dy}{dx} = 10 + 3y - y^2$.

3 a) Find the critical points and phase portrait, and then classify each critical point in terms of its stability.

To find the critical points; $f(y) = 0 \Rightarrow 10 + 3y - y^2 = 0$

$\Rightarrow (5-y)(2+y) = 0$

$\Rightarrow y = 5$, $y = -2$ are critical pts.



$y = -2$ is unstable (repeller).

$y = 5$ is asymptotically stable (attractor).

b) Sketch a typical solution curve determined by the graph of the equilibrium solution..

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