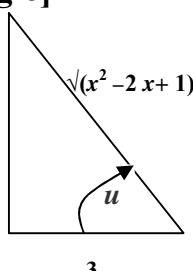


Integration Formulas (Required for MATH 202)

No	Type	Formula	Exercise I	Exercise II
1	Power Rule	$\int (ax+b)^m dx = \frac{(ax+b)^{m+1}}{a(m+1)} + c; m \neq -1$	$\int (2-7x)^{11} dx$	$\int \frac{1}{(-8x+45)^{21}} dx$
2.	Substitution	$\int g(f(x))f'(x)dx = \int g(u)du$ where $u = f(x)$	$\int \frac{(x-2x^3)dx}{6(x^4-x^2+7)^{\frac{7}{3}}}$	$\int (x^2-6x)(x^3-9x^2-17)dx$
3	Direct Formulas exp & log	i. $\int \frac{1}{x} dx = \ln x + c$ ii. $\int e^x dx = e^x + c$ iii. $\int a^x dx = \frac{a^x}{\ln a} + c; a > 0$	i. $\int \frac{18}{20x-7} dx$ ii. $\int -71e^{4x+9} dx$ iii. $\int 5(2.5)^{-4x+3} dx$	i. $\int \frac{3x^2-7x}{2x^3-7x^2+5} dx$ ii. $\int (3-8x^3)e^{-2x^4+3x} dx$ iii. $\int \frac{5(6x-7)}{7^{3x^2-7x}} dx$
4	Simple Trig. Function	i. $\int \sin(ax+b)dx = \frac{-1}{a} \cos(ax+b) + c$ ii. $\int \cos(ax+b)dx = \frac{1}{a} \sin(ax+b) + c$ iii. $\int \sec^2 x dx = \tan x + c$ iv. $\int \csc^2 x dx = -\cot x + c$ v. $\int \tan x \sec x dx = \sec x + c$ vi. $\int \cot x \csc x dx = -\csc x + c$	i. $\int \sin(6x-4)dx$ ii. $\int \cos(8-9x)dx$ iii. $\int \sec^2(3x+7)dx$ iv. $\int \csc^2(2-5x)dx$ v. $\int \tan 4x \sec 4x dx$ vi. $\int \cot \pi x \csc \pi x dx$	i. $\int x^4 \sin(6x^5-4)dx$ ii. $\int \sin^3 5x \cos 5x dx$ iii. $\int \tan^4 8x \sec^2 8x dx$ iv. $\int \cot^3 x \csc^4 x dx$ v. $\int \frac{1}{x^2} \tan \frac{\pi}{x} \sec \frac{\pi}{x} dx$ vi. $\int x^6 \cot x^7 \csc x^7 dx$
5	Use of Inverse Trig. Functions Also, see 9	i. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$ ii. $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$ iii. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$	i. $\int \frac{dx}{\sqrt{25-9x^2}}$ ii. $\int \frac{dx}{100+49x^2}$ iii. $\int \frac{dx}{x\sqrt{16x^2-81}}$	i. $\int \frac{dx}{\sqrt{18+8x-x^2}}$ ii. $\int \frac{dx}{170+70x+49x^2}$ iii. $\int \frac{dx}{(x-9)\sqrt{x^2-18x}}$
5	Integration by Parts	Rule: $\int u dv = uv - \int v du$ i. $\int xe^x dx = xe^x - e^x + c;$ $[u = x, dv = e^x dx \Rightarrow du = dx, v = \int e^x dx]$ ii. $\int x \sin x dx = -x \cos x + \sin x + c$ iii. $\int \ln x dx = x \ln x - x + c;$ $[u = \ln x, dv = dx]$ iv. $\int \cos xe^x dx = \frac{1}{2}(\cos xe^x + \sin xe^x) + c$	i. $\int \sin^{-1} x dx$ ii. $\int \log x dx$ iii. $\int x^2 \ln x dx$ iv. $\int x \tan^{-1} x dx$ v. $\int x^2 2^x dx$ vi. $\int x^2 \sin x dx$ vii. $\int x^3 \sin x^2 dx$	i. $\int 6xe^{5x-9} dx$ ii. $\int x^7 \cos x^4 dx$ iii. $\int x^4 \ln(3x^5-11) dx$ iv. $\int \frac{\ln x^2}{x^2} dx$ v. $\int (x^2-2x)\cos^{-1}(x^3-3x^2) dx$ vi. $\int x \sec^{-1} x dx$ vii. $\int x \tan^{-1} x dx$

6	More Trig. Formulas	i. $\int \tan x dx = -\ln \cos x + c$	i. $\int x \tan(x^2 - 7) dx$	i. $\int (5x-1) \sec(5x^2 - 2x) dx$
		ii. $\int \cot x dx = \ln \sin x + c$	ii. $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$	ii. $\int \frac{1}{(3x-1)^2} \csc \frac{1}{3x-1} dx$
		iii. $\int \sec x dx = \ln \sec x + \tan x + c$	iii. $\int x \sec x dx$	
		iv. $\int \csc x dx = \ln \csc x - \cot x + c$		

Use of Trig. Identities in Integration

	Trig. Identities	Exercises		
7	i. $\cos^2 x = \frac{1 + \cos 2x}{2}$	i. $\int \sin^2(3x+1) dx$	ii. $\int \sin^4 5x dx$	iii. $\int x \sin^2(7x^2 + 3) dx$
	ii. $\sin^2 x = \frac{1 - \cos 2x}{2}$ (See **)	iv. $\int x \cos^2(3x+1) dx$	v. $\int \cos^3 5x dx$	vi. $\int x \cos(7x^2 + 3) dx$
	iii. $\sin 2x = 2 \sin x \cos x$	vii. $\int \cos^2 3x \sin^2 3x dx$	viii. $\int x^3 \sin^2(x^2 + 3) dx$	
8	i. $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$; ii. $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$; iii. $\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$			
	iv. $\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$; v. $\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$			
	i. $\int \sin 5x \cos 3x dx$	ii. $\int \cos 5x \cos 3x dx$	iii. $\int \sin 5x \sin 3x dx$	
9	Example: [Completion of Square & Use of Right Triangle]			
	$I = \int \frac{x-2}{\sqrt{x^2-2x+10}} dx : x^2 - 2x + 10 = (x-1)^2 + 9; \text{ Set } x-1 = 3 \tan u \Rightarrow dx = 3 \sec^2 u du.$			
$I = \int (3 \tan u - 1) \sec u du = 3 \sec u - \ln \sec u + \tan u + c = \sqrt{x^2 - 2x + 10} - \ln \left \frac{\sqrt{x^2 - 2x + 10}}{3} + \frac{x-1}{3} \right + c$				

Use of Partial Fractions Decomposition of Rational Functions $N(x)/D(x)$ in Integration

[*Note: If $\deg(N) \geq \deg(D)$, first divide N by D . Then find Partial Fractions Decomposition.]

Here, in (i-iv), we assume that **deg** (Numerator) < **deg** (Denominator)

10	Type	Method	Exercises
i	Non-Repeated Linear Factors	$\frac{p(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$ <p>[Write: $p(x) = A(x-b) + B(x-a)$]</p>	$\frac{x+11}{x^2-2x-15} = \frac{A}{x+3} + \frac{B}{x-5};$ <p>Ans: $A = -1, B = 2$</p>
ii	Repeated Linear Factors	$\frac{p(x)}{(x-a)(x-b)^2} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2}$	$\frac{x^2+2x+7}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2};$ <p>Ans: $A = 7, B = -6, C = 10$</p>
iii	Non-Repeated Non-Linear Factors	$\frac{p(x)}{(x^2+bx+c)(x^2+dx+g)} = \frac{Ax+B}{x^2+bx+c} + \frac{Cx+D}{x^2+dx+g}$	$\frac{3x+16}{(x-2)(x^2+7)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+7};$ <p>Ans: $A = 2, B = -2, C = -1$</p>
iv	Repeated Non-Linear Factors	$\frac{p(x)}{(x-a)(x^2+bx+c)^2} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c} + \frac{Dx+E}{(x^2+bx+c)^2}$	$\frac{4x^3+5x^2+7x-1}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2};$ <p>Ans: $A = 4, B = 1, C = 2, D = -2$</p>

* v	deg (N) ≥ deg (D)	$\frac{N(x)}{D(x)} = \frac{N(x)}{(x-a)(x-b)} = Q(x) + \frac{R(x)}{(x-a)(x-b)};$ <p>where $\frac{R(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$</p>	$\frac{x^3-4x^2-19x-35}{x^2-7x} = Q(x) + \frac{R(x)}{x^2-7x};$ <p>where $\frac{R(x)}{x^2-7x} = \frac{A}{x} + \frac{B}{x-7}$; Ans: $Q(x) = x + 3$; $R(x) = 2x - 35$; $A = 4, B = 1, C = 2, D = -2$</p>
**	Trig. Identities: $\cos^2 \theta + \sin^2 \theta = 1$; $1 + \tan^2 \theta = \sec^2 \theta$; $1 + \cot^2 \theta = \csc^2 \theta$		