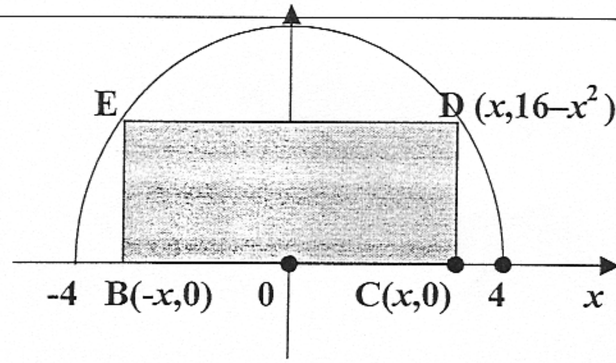
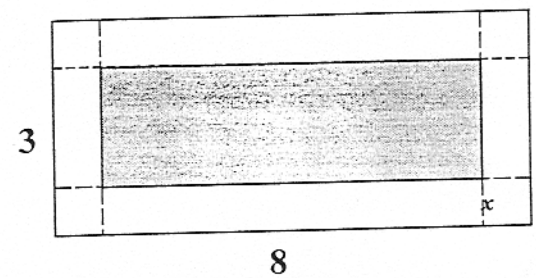
Q 8. A rectangle has its 2 lower corners on the x-axis and 2 upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

Solution:

<table>
<tr><td>1</td><td>Draw the Diagram:
{Sketch the cuve $y = 16 - x^2$ in xy-plane and place the Rectangle BCDE inside the curve as required in Q 8.}
Note: Midpoint of the base of Rectangle must be on the Origin.</td><td>
</td></tr>
<tr><td>2</td><td>Identify & Label the Unknowns</td><td>• Coordinates of Four corners of Rectangle:
B = (-x,0)   C = (x,0)
D = $(x, 16 - x^2)$ E = $(-x, 16 - x^2)$
• Rectangle's: Length = L ; Width = W
• Area of Rectangle = A</td></tr>
<tr><td>3</td><td>Identify the Variable to be Optimized</td><td>A</td></tr>
<tr><td>4</td><td>Relation among the Variables</td><td>A = L W</td></tr>
<tr><td>5</td><td>Side Conditions (See Fig.)</td><td>$L = 2x$; $W = 16 - x^2$; $0 \le x \le 4$;</td></tr>
<tr><td>6</td><td>Convert Main Equation To Equation of One Variable</td><td>$A = L\,W = 2x(16 - x^2)$
$= 32x - 2x^3$</td></tr>
<tr><td>7</td><td>Find C.N. of A</td><td>$dA/dx = 32 - 6x^2 = 0$; $x = 4/\sqrt{3}$</td></tr>
<tr><td>8</td><td>Use 2nd Deriv. To test C.N</td><td>$A'' = -4x \Rightarrow A''\left(4/\sqrt{3}\right) < 0 \Rightarrow$ Maxim at C.N.*</td></tr>
<tr><td>9</td><td>Test the C.N. and 0 , 4 for Max/Mini Values</td><td>
<table>
<tr><td>x</td><td>L</td><td>W</td><td>A</td><td><u>A will be</u></td></tr>
<tr><td>0</td><td>0</td><td>16</td><td>0</td><td></td></tr>
<tr><td>$4/\sqrt{3}$</td><td>$8/\sqrt{3}$</td><td>$32/3$</td><td>$256/3\sqrt{3}$</td><td>Maxim*</td></tr>
<tr><td>4</td><td>8</td><td>0</td><td>0</td><td></td></tr>
</table>
</td></tr>
</table>

Q 19. An open box is to be made from a 3 ft by 8 ft rectangular piece of sheet metal by cutting out squares of equal size from the 4 corners and bending up the sides. Find the maximum volume that the box can have.

Solution:

1	**Draw the Diagram.** **[From each corner of the Rectangle, cut a square of side length x]**	
2	**Identify & Label the Unknowns**	• **Side Length of Removed Square = x** • **Box: Length= L ; Width=W ; Height = H** • **Volume of Box = V**
3	**Identify the Variable to be Optimized**	**V**
4	**Relation among the Variables**	**V = L W H**
5	**Side Conditions (See Fig.)**	**L= 8-2x ; W = 3-2x ; H = x ;** $0 \le x \le 3/2$
6	**Convert Main Equation To Equation of One Variable**	**V = L W H = $x(8-2x)(3-2x)$** **$= 4x^3 - 22x^2 + 24x$**
7	**Find C.N. of V**	**$dV/dx = 12x^2 - 44x + 24 = 4(3x-2)(x-3) = 0$** **C.N : $x = 2/3$; [Note: $x = 3$ does not satisfy (5)]**
8	**Use 2nd Deriv. To test C.N**	**$V'' = 24x - 44$; $V''(2/3) < 0$ [Maxim at C.N.]***
9	**Test the C.N. and 0 , 4 for Max/Mini Values**	

x	L	W	H	V	*V* **will be**
0	**8**	**3**	**0**	**0**	
2/3	**20/3**	**5/3**	**2/3**	**200/27**	**Maxim***
3/2	**5**	**0**	**3/2**	**0**	