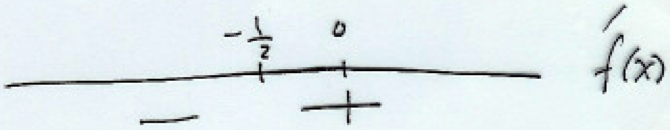
Example 6 Sketch $y = 6x^{\frac{1}{3}} + 3x^{\frac{4}{3}}$

Sol. $y = 3x^{\frac{1}{3}}(2+x)$

- $D = \mathbb{R}$
- Intercepts: $\boxed{(0,0),\ (-2,0)}$
- No symmetry
- Asymptotes: None
- Increase-decrease: $f'(x) = 3x^{\frac{1}{3}} + x^{-\frac{2}{3}}(2+x)$

$$= x^{-\frac{2}{3}}[3x+2+x] = x^{-\frac{2}{3}}(4x+2) =$$

$\therefore\ f'(x) = \dfrac{2(2x+1)}{x^{\frac{2}{3}}}$



$f'(x) = 0 \Rightarrow x = -\frac{1}{2}$

$x = -\frac{1}{2}, 0$ are critical pts

$\uparrow\ (-\frac{1}{2}, 0) \cup (0, \infty)$

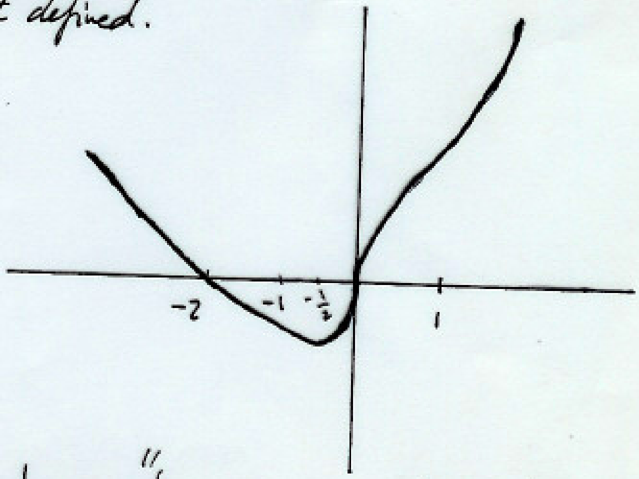
$\downarrow\ (-\infty, -\frac{1}{2})$

- Local extrema: from the sign of f' $\Rightarrow$ $\boxed{x = -\frac{1}{2}}$ is a relative min.
- Vertical tangency: Note that $f'(0)$ is not defined.

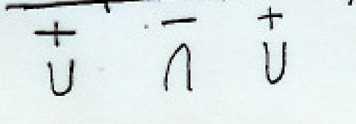
$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \dfrac{2(2x+1)}{x^{\frac{2}{3}}} = +\infty$

$\lim_{x \to 0^-} f'(x) = \qquad = +\infty$

$\Rightarrow$ Vertical tangency at $x = 0$



- Concavity: $f''(x) = \dfrac{4(x-1)}{3x^{\frac{5}{3}}}$



$x = 0, 1$ are inflection pts