Example 5 Sketch $y = (x-4)^{\frac{2}{3}}$

al.

- $D = \mathbb{R}$
- intercepts: $x=0 \Rightarrow y = \sqrt[3]{16}$, $y=0 \Rightarrow x=4$ $\Big\} \Longrightarrow \boxed{(0, \sqrt[3]{16}),\ (4, 0)}$
- No Symmetry
- Asymptotes: None. $\lim\limits_{x \to \pm\infty} f(x) = +\infty$
- increase-decrease: $f'(x) = \dfrac{2}{3(x-4)^{\frac{1}{3}}}$

  $\uparrow$ $(4, \infty)$

  $\downarrow$ $(-\infty, 4)$

  $-$ 4 $+$ $f'(x)$

- Extrema: Using 1st derivative test; [We can not use the second der. Test, why?]

  Note that $x=4$ is a critical point since $f'(4)$ is not defined.

  From the sign of $f' \Longrightarrow$ $x=4$ is a relative min i.e. $\boxed{(4,0)}$ Min.

- Concavity: $f''(x) = -\dfrac{2}{9(x-4)^{\frac{4}{3}}} = \dfrac{-2}{9(\sqrt[3]{x-4})^4} < 0 \quad \forall x, x \neq 4$

  $\therefore$ The curve is concave down on $(-\infty, 4) \cup (4, \infty)$

- Vertical tangent line: Note that $f'(4)$ does not exist

  $\lim\limits_{x \to 4^+} f'(x) = \lim\limits_{x \to 4^+} \dfrac{2}{3(x-4)^{\frac{1}{3}}} = +\infty$

  $\lim\limits_{x \to 4^-} f'(x) = -\infty$

  $\Rightarrow$ There is a vertical tangent line and a cusp at $x=4$