

# Example 5 Sketch $y = (x-4)^{\frac{2}{3}}$

sol.

$D = \mathbb{R}$

Intercepts:  $x=0 \Rightarrow y = \sqrt[3]{16}$   
 $y=0 \Rightarrow x=4$  }  $\Rightarrow$   $(0, \sqrt[3]{16}), (4, 0)$

No Symmetry

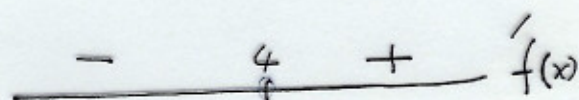
Asymptotes: None.

$\lim_{x \rightarrow \pm\infty} f(x) = \infty$

Increase-decrease:  $f'(x) = \frac{2}{3(x-4)^{\frac{1}{3}}}$

$\uparrow (4, \infty)$

$\downarrow (-\infty, 4)$



Extrema: Using 1st derivative test: [We can not use the second der. Test, Why?]

Note that  $x=4$  is a critical point since  $f'(4)$  is not defined.

From the sign of  $f' \Rightarrow x=4$  is a relative min i.e.  $(4, 0)$  Min.

Concavity:  $f''(x) = \frac{-2}{9(x-4)^{\frac{4}{3}}} = \frac{-2}{9(\sqrt[3]{x-4})^4} < 0 \quad \forall x, x \neq 4$

$\therefore$  The curve is concave down on  $(-\infty, 4) \cup (4, \infty)$

Vertical tangent line: Note that  $f'(4)$  does not exist

$\lim_{x \rightarrow 4^+} f'(x) = \lim_{x \rightarrow 4^+} \frac{2}{3(x-4)^{\frac{1}{3}}} = +\infty$

$\lim_{x \rightarrow 4^-} f'(x) = -\infty$

$\Rightarrow$  There is a vertical tangent line and a cusp at  $x=4$

