Ex 3 Sketch the graph of

$$y = \frac{x^2-9}{2x-4}$$

Sol.

* $D = \mathbb{R} - \{2\} \qquad = (-\infty, 2) \cup (2, \infty)$

* Intercepts: $\left.\begin{array}{l} x=0 \Rightarrow y = 9/4 \\ y=0 \Rightarrow x = \pm 3 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l}(0, \frac{9}{4}) \\ (3,0),\ (-3,0)\end{array}}$

* No symmetry:

* increase_decrease:

$$f'(x) = \frac{(2x-4)2x-(x^2-9)2}{(2x-4)^2} = \frac{4x^2-8x-2x^2+18}{(2x-4)^2}$$

$$= \frac{2x^2-8x+18}{(2x-4)^2} = \frac{2(x^2-4x+9)}{4(x-2)^2}$$

$$= \frac{x^2-4x+9}{2(x-2)^2}$$

$f'(x) > 0 \quad \forall x \in D(f) \Rightarrow \boxed{f \text{ is increasing}}$

$f'(x) \neq 0$

$\Rightarrow \boxed{\text{No relative extrema}}$

* Concavity: $f''(x) = \dfrac{2(x-2)^2(2x-4)-(x^2-4x+9)4(x-2)}{4(x-2)^4}$

$$= \frac{4(x-2)\left[(x-2)^2 - x^2 + 4x - 9\right]}{4(x-2)^4}$$

$$f''(x) = -\frac{5}{(x-2)^3}$$

$$\begin{array}{ccc} + & 2 & - \\ \hline \cup & & \cap \end{array} \quad f''$$

$f'' \neq 0$

* *Asymptotes*

- Clearly $x = 2$ is a V. A.
- No horizontal asymptote.
- Since $\deg(x^2-9) = 1 + \deg(2x-4) \Rightarrow$ there is an oblique asymptote:

$y = \frac{1}{2}x + 1$ is an oblique asymptote