Example 2 Sketch $y = \frac{x^2-1}{x^3}$

Sol. • $D = \mathbb{R} - \{0\}$

• **Intercepts:** x-intercepts $\boxed{(-1,0), (1,0)}$. No y-intercept

• **Symmetry:** $f(-x) = -\frac{x^2-1}{x^3} = -f(x)$. $\Rightarrow$ the graph is Sym. about $(0,0)$.

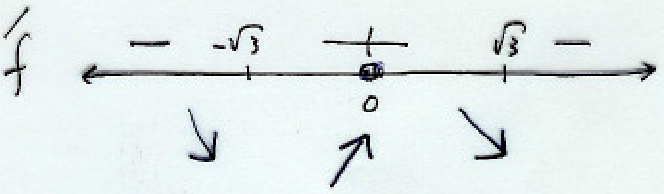
• **Asymptotes:** $\boxed{x=0 \text{ V.A.}}$, $\boxed{y=0 \text{ H.A.}}$ since $\lim\limits_{x\to\pm\infty} f(x) = 0$

• **Increase-decrease:** $f'(x) = \frac{3-x^2}{x^4}$ $\qquad f'(0)$ not defined.

$f'(x) = 0 \Rightarrow x = \pm\sqrt{3}$. Critical pts are $\boxed{\sqrt{3}, -\sqrt{3}, 0}$

$\uparrow$ $(-\sqrt{3}, \sqrt{3})$

$\downarrow$ $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$



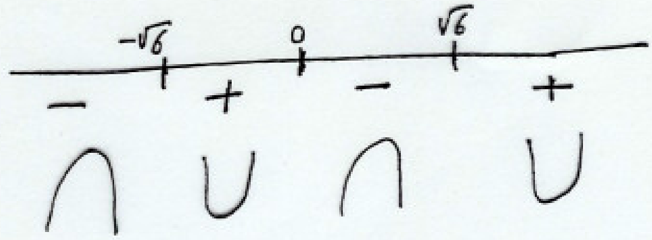
• **Local extrema:** $f''(x) = \frac{2(x^2-6)}{x^5}$

$f''(\sqrt{3}) = \frac{-2\sqrt{3}}{3} < 0 \Rightarrow$ relative max at $x = \sqrt{3}$ i.e $\boxed{\left(\sqrt{3}, \frac{2\sqrt{3}}{9}\right)}$ Max $\approx (1.73, .38)$

$f''(-\sqrt{3}) = \frac{2\sqrt{3}}{3} > 0 \Rightarrow$ relative min at $x = -\sqrt{3}$ i.e $\boxed{\left(-\sqrt{3}, \frac{-2\sqrt{3}}{9}\right)}$ Min $\approx (-1.73, -.38)$

- **Concavity:** $f''(x) = 0 \Rightarrow \dfrac{2(x^2-6)}{x^5} = 0$

$$\Rightarrow x = \pm\sqrt{6}$$



- **Inflection points** $x = -\sqrt{6}$, $x = \sqrt{6}$. No inflection point at $x=0$ (b

- **Additional points:** $\left(2, \frac{3}{8}\right)$, $\left(-2, -\frac{3}{8}\right)$.