Ex 1 Sketch the graph of $f(x) = x^5 - 5x^3$

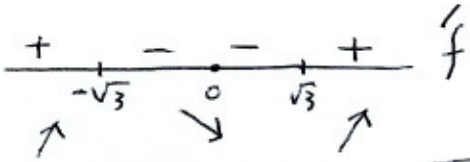
Sol.

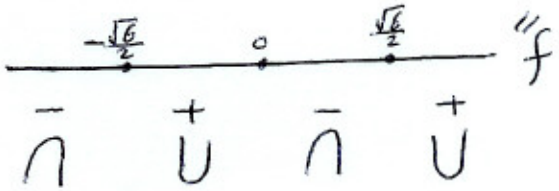
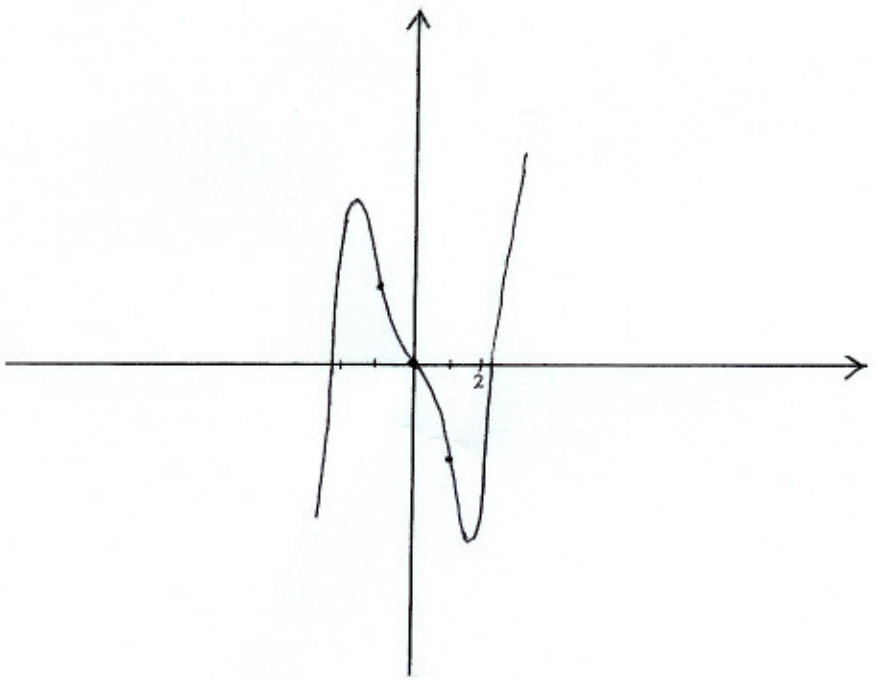
* $D = \mathbb{R}$ Remember $f(x)$ is a polynomial and in this case $\lim_{x\to+\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = -\infty$

* **Intercepts:** $x = 0 \Rightarrow y = 0$
  $y = 0 \Rightarrow x^3(x^2-5) = 0 \Rightarrow x = 0$ (multiplicity 3), $x = \pm\sqrt{5}$
  $\therefore$ $(0,0)$, $(\sqrt{5},0)$, $(-\sqrt{5},0)$

* **Symmetry:** The graph is symmetric about $(0,0)$
  Since $f(-x) = -f(x)$

* **Increase-decrease:** $f'(x) = 5x^4 - 15x^2 = 5x^2(x^2-3)$
  $f'(x) = 0 \Rightarrow x = 0, \sqrt{3}, -\sqrt{3}$ ← Critical points



* **Relative extrema:** Using 1st derivative test $\Rightarrow$ $f(x)$ has a relative max at $x = -\sqrt{3}$ $(-\sqrt{3}, 6\sqrt{3})$
  and a relative min at $x = \sqrt{3}$ $(\sqrt{3}, -6\sqrt{3})$

* **Concavity:** $f''(x) = 20x^3 - 30x = 10x(2x^2-3)$
  $f''(x) = 0 \Rightarrow x = 0, \frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}$
  Inflection points: