Guideline to Solve Applied Maxima / Minma Problems

1. **Draw a diagram suitable for the Problem.**
2. **Identify the known and unknown quantities.**
3. **Label the unknown quantities.**
4. **Identify the variable (in Step 3) that is to be Maximized or Minimized.**
5. **Find the Relation among the Variables / Known Quantities of the problem**
6. **Find the Additional /Side conditions, if any.**
7. **Convert the Main Equation to Equation of 1 variable using Step 6, if required**
8. **Find the interval suitable for the variable to be Maximized or Minimized.**
9. **Find the Critical Numbers of the function in Step 7.**
10. **Check the C.N. (in Step 9) and also the end points of the Interval (in Step 8) for Relative Extremum.**

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Ex 5.3

Q 2. How should 2 nonnegative numbers be chosen so that their sum is 1 and sum of their squares is (a) as small as possible (b) as large as possible.

Solution:

1. **Identify the Unknowns:**	**2 nonnegative numbers; Sum of Squares of x & y**
2. **Label the Unknowns:**	**x & y =2 nonnegative integers;** **z = Sum of Squares of x & y**
3. **Identify the Variable to be Maximized/Minimized:**	**z**
4. **Relation among the Variables**	
5. **Side Conditions:**	**$x + y = 1$**
6. **Convert Main Eq. To Eq. of One Variable:**	**$z = x^2 + (1- x)^2$**

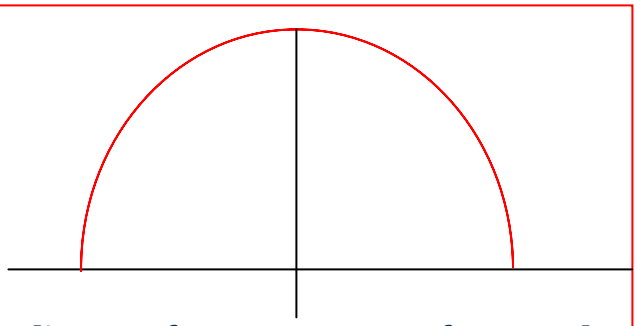
Interval for x & y
[0, 1]

- **Find C.N of z**
- **Test the C.N. and 0,1 for Max/Mini Values**

z is Minimum: $x = y = \frac{1}{2}$
z is Maximum: $x = 1$, $y = 0$

Q 8. A rectangle has its 2 lower corners on the *x*-axis and 2 upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

1. Draw the Diagram:



2. Identify the Unknowns:

i.Coordinates of upper corners of rectangle
ii. Length & Width of Rectangle
iii. Area of Rectangle

3. Label the Unknowns:

B = (-*x*,0) **C = (*x*,0)**
D = (x, $16 - x^2$) **E = ($-x$, $16 - x^2$)**
L = 2*x* **W = $16 - x^2$**
A=Area of Rectangle

4. Identify the Variable to be Maximized/Minimized:

A

5. Relation among the Variables

A = L W

6. Side Conditions:

$0 \le x \le 4$

7. Convert Main Eq. To Eq. of One Variable:

A = L W

= $2x(16 - x^2)$

= $32x - 2x^3$

A'= $32 - 6x^2$

C.N:

$x = \frac{4}{\sqrt{3}}$

A''= $-12x$

Interval for *x* & *y*
[0, 4]

- **Find C.N of *A***
- **Test the C.N. and 0,4 for Max/Mini values**

A is Max. at $x = \frac{4}{\sqrt{3}}$

$\Rightarrow L = 2x = \frac{8}{\sqrt{3}}$

$W = 16 - x^2 = \frac{32}{3}$

Classroom Problems

Q 19. An open box is to be made from a 3 ft by 8 ft rectangular piece of sheet metal by cutting out squares of equal size from the 4 corners and bending up the sides. Find the maximum volume that the box can have.

Q 26. Show that the right circular cylinder of greatest volume that can be inscribed in a right circular cone has volume that is 4/9 the volume of the cone.

Q 38. A trapezoid is inscribed in a semicircle of radius 2 so that one side is along the diameter. Find the maximum possible area of the trapezoid.

Q F. Find all points on the curve $x^2 - y^2 = 1$ closest to (0,2).