Math 101 – Quiz # 6

Name: Solution **Serial#:**___________

Let $f(x) = x(1-x)^{\frac{2}{5}}$.

1. Find the critical points of f.

$$f'(x) = x\left[\tfrac{2}{5}(1-x)^{-\frac{3}{5}}(-1)\right] + (1-x)^{\frac{2}{5}}$$

$$= \frac{-2x}{5(1-x)^{\frac{3}{5}}} + (1-x)^{\frac{2}{5}} = \frac{-2x}{5(1-x)^{\frac{3}{5}}} + \frac{5(1-x)}{5(1-x)^{\frac{3}{5}}}$$

$$= \frac{-2x+5-5x}{5(1-x)^{\frac{3}{5}}} = \frac{5-7x}{5(1-x)^{\frac{3}{5}}}$$

$$\left.\begin{array}{l} f'(x) = 0 \Rightarrow x = \frac{5}{7} \\ \text{Also, } f'(x) \text{ DNE at } x = 1 \end{array}\right\} \Longrightarrow \text{Critical points are } x = \tfrac{5}{7} \text{ and } x = 1$$

2. Find the intervals on which f is increasing and on which f is decreasing.

We check the sign of f' to conclude:

	$(-\infty, \frac{5}{7})$	$(\frac{5}{7}, 1)$	$(1, \infty)$
$5-7x$	+	−	−
$5(1-x)^{\frac{3}{5}}$	+	+	−
$f'(x)$	+	−	+
	↗	↘	↗

$f(x)$ is increasing on $(-\infty, \frac{5}{7}) \cup (1, \infty)$

decreasing on $(\frac{5}{7}, 1)$

3. f has a stationary point at $x = \frac{5}{7}$

4. Locate the relative extrema of f.

From parts 1 and 2 above, we can see that f has:

a relative max at $x = \frac{5}{7}$ and a relative min at $x = 1$.

5. Discuss the concavity of f.

$$f''(x) = \frac{-35(1-x)^{\frac{3}{5}} + (5-7x)5(1-x)^{-\frac{2}{5}}}{25(1-x)^{\frac{6}{5}}} = \frac{5(1-x)^{-\frac{2}{5}}\left[-7(1-x)+5-7x\right]}{25(1-x)^{\frac{6}{5}}}$$

$$= \frac{-2}{5(1-x)^{\frac{8}{5}}} < 0 \quad \forall x \neq 1.$$

$\therefore$ $f(x)$ is concave down on $(-\infty, 1) \cup (1, \infty)$