

Math 101 - Quiz # 6

Name: Solution

Serial#: _____

Let $f(x) = x(1-x)^{\frac{2}{5}}$.

1. Find the critical points of f .

$$\begin{aligned} f'(x) &= x \left[\frac{2}{5} (1-x)^{-\frac{3}{5}} (-1) \right] + (1-x)^{\frac{2}{5}} \\ &= \frac{-2x}{5(1-x)^{\frac{3}{5}}} + (1-x)^{\frac{2}{5}} = \frac{-2x}{5(1-x)^{\frac{3}{5}}} + \frac{5(1-x)}{5(1-x)^{\frac{3}{5}}} \\ &= \frac{-2x+5-5x}{5(1-x)^{\frac{3}{5}}} = \frac{5-7x}{5(1-x)^{\frac{3}{5}}} \end{aligned}$$

$$\left. \begin{aligned} f'(x) = 0 &\Rightarrow x = \frac{5}{7} \\ \text{Also, } f'(x) \text{ DNE at } x &= 1 \end{aligned} \right\} \Rightarrow \text{Critical points are } x = \frac{5}{7} \text{ and } x = 1$$

2. Find the intervals on which f is increasing and on which f is decreasing.

We check the sign of f' to conclude:

$f(x)$ is increasing on $(-\infty, \frac{5}{7}) \cup (1, \infty)$

decreasing on $(\frac{5}{7}, 1)$

	$\frac{5}{7}$	1	
$5-7x$	+	-	-
$5(1-x)^{\frac{3}{5}}$	+	+	-
$f'(x)$	+	-	+
	↑	↓	↑

3. f has a stationary point at $x = \frac{5}{7}$

4. Locate the relative extrema of f .

From parts 1 and 2 above, we can see that f has:

a relative max at $x = \frac{5}{7}$ and a relative min at $x = 1$.

5. Discuss the concavity of f .

$$\begin{aligned} f''(x) &= \frac{-35(1-x)^{\frac{2}{5}} + (5-7x)5(1-x)^{-\frac{2}{5}}}{25(1-x)^{\frac{6}{5}}} = \frac{5(1-x)^{-\frac{2}{5}}[-7(1-x)+5-7x]}{25(1-x)^{\frac{6}{5}}} \\ &= \frac{-2}{5(1-x)^{\frac{6}{5}}} < 0 \quad \forall x \neq 1. \end{aligned}$$

$\therefore f(x)$ is concave down on $(-\infty, 1) \cup (1, \infty)$