

King Fahd University of Petroleum and Minerals
 Department of Mathematical Sciences
 Math 101 Quiz # 5 Fall (051)

ID #: _____ NAME: Solution
 Serial # _____ Section #: _____

Find y' for each of the following:

1. $y = \ln(7x\sqrt{3-x^2})$.

$$y = \ln 7x + \ln \sqrt{3-x^2} = \ln 7x + \frac{1}{2} \ln(3-x^2)$$

$$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{-2x}{3-x^2} = \frac{1}{x} - \frac{x}{3-x^2} = \frac{3-4x^2}{x(3-x^2)}$$

2. $\sin^{-1}(\sqrt{xy}) = y$.

Differentiating implicitly, we have $\frac{1}{\sqrt{1-(\sqrt{xy})^2}} \cdot \frac{1}{2\sqrt{xy}} [x y' + y] = y'$

$$y' [2\sqrt{xy} \sqrt{1-xy}] = x y' + y$$

$$y' [2\sqrt{xy} \sqrt{1-xy} - x] = y \Rightarrow y' = \frac{y}{2\sqrt{xy} \sqrt{1-xy} - x} = \frac{y}{2\sqrt{xy-x^2}-x}$$

3. $y = 3x \tan^{-1} e^{2x}$.

$$y' = 3x \cdot \frac{1}{1+(e^{2x})^2} \cdot 2e^{2x} + 3 \tan^{-1} e^{2x}$$

$$= \frac{6x e^{2x}}{1+e^{4x}} + 3 \tan^{-1} e^{2x}$$

4. $y = x^{\sqrt{x}} \Rightarrow \ln y = \sqrt{x} \ln x$

$$y \cdot \frac{1}{y} = \sqrt{x} \cdot \frac{1}{x} + (\ln x) \frac{1}{2\sqrt{x}}$$

$$y' = y \left(\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} \right) = x^{\sqrt{x}} \left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}} \right) = x^{\sqrt{x}} \left(\frac{2 + \ln x}{2\sqrt{x}} \right)$$