Math 101 Quiz # 2

Name: Solution I.D. # ____________ Section # ____ Serial #

1. Find each of the following limits:

i) $\lim_{x\to 0}\frac{\tan 5x}{\sin 3x}$

$$= \lim_{x\to 0} \frac{\frac{\tan 5x}{x}}{\frac{\sin 3x}{x}} = \lim_{x\to 0} \frac{5\,\frac{\tan 5x}{5x}}{3\,\frac{\sin 3x}{3x}} = \frac{5}{3}\lim_{x\to 0}\frac{\frac{\tan 5x}{5x}}{\frac{\sin 3x}{3}} = \frac{5}{3}$$

ii) $\lim_{\theta\to 0}\frac{7}{\theta\csc\theta}$

$$= \lim_{\theta\to 0}\frac{7}{\frac{\theta}{\sin\theta}} = \lim_{\theta\to 0}\frac{7\sin\theta}{\theta} = 7\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 7(1) = 7.$$

iii) $\lim_{x\to 2}\frac{\sin \pi x}{x-2}$ (Hint: use the substitution $x-2=t$)

Put $x-2=t$ then $x=t+2$ and $t\to 0$ as $x\to 2$.

$$\lim_{x\to 2}\frac{\sin\pi x}{x-2} = \lim_{t\to 0}\frac{\sin\pi(t+2)}{t} = \lim_{t\to 0}\frac{\sin(\pi t+2\pi)}{t} = \lim_{t\to 0}\frac{\sin\pi t}{t}$$

$$= \pi\lim_{t\to 0}\frac{\sin\pi t}{\pi t} = \pi(1) = \pi.$$

2. Let $f(x)=\frac{1}{x-2}$

i) Find the average rate of change of y with respect to x over the interval [3, 5].

$$r_{ave} = \frac{f(5)-f(3)}{5-3} = \frac{\frac{1}{5-2}-\frac{1}{3-2}}{5-3} = \frac{\frac{1}{3}-1}{2} = -\frac{1}{3}$$

ii) Find the instantaneous rate of change of y with respect to x at $x=3$

$$r_{inst} = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h} = \lim_{h\to 0}\frac{\frac{1}{3+h-2}-\frac{1}{3-2}}{h} = \lim_{h\to 0}\frac{\frac{1}{h+1}-1}{h}$$

$$= \lim_{h\to 0}\frac{1-h-1}{h(h+1)} = \lim_{h\to 0}\frac{-h}{h(h+1)} = \lim_{h\to 0}\frac{-1}{h+1} = -1$$