

Note: Show all work. *Solution* No Calculator is allowed.

Evaluate the limits in Questions 1-5

1.  $\lim_{t \rightarrow 0} \frac{\sqrt{t+7} - \sqrt{7}}{t} =$  (6 pts)

$$= \lim_{t \rightarrow 0} \frac{\sqrt{t+7} - \sqrt{7}}{t} \cdot \frac{\sqrt{t+7} + \sqrt{7}}{\sqrt{t+7} + \sqrt{7}}$$

$$= \lim_{t \rightarrow 0} \frac{t+7-7}{t(\sqrt{t+7} + \sqrt{7})} = \lim_{t \rightarrow 0} \frac{\cancel{t}}{\cancel{t}(\sqrt{t+7} + \sqrt{7})}$$

$$= \lim_{t \rightarrow 0} \frac{1}{\sqrt{t+7} + \sqrt{7}}$$

$$= \frac{1}{2\sqrt{7}}$$

2.  $\lim_{x \rightarrow \infty} \frac{\sqrt[4]{16x^8 - 809x + 10}}{6x^2 - 1}$  (6 pts)

$$= \lim_{x \rightarrow \infty} \frac{\sqrt[4]{\frac{16x^8}{x^8} - \frac{809x}{x^8} + \frac{10}{x^8}}}{\frac{6x^2}{x^2} - \frac{1}{x^2}}$$

$[\sqrt[4]{x^8} = x^2]$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt[4]{16 - \frac{809}{x^7} + \frac{10}{x^8}}}{6 - \frac{1}{x^2}} = \frac{2}{6}$$

$$= \frac{1}{3}$$

$$3. \quad \lim_{x \rightarrow 3^-} \frac{3x-9}{|3-x|} =$$

(5 pts)

$$= \lim_{x \rightarrow 3^-} \frac{3(x-3)}{|3-x|}$$

$$= \lim_{x \rightarrow 3^-} \frac{3(x-3)}{3-x} = \lim_{x \rightarrow 3^-} \frac{-3(3-x)}{3-x}$$

$$= -3$$

$$4. \quad \lim_{x \rightarrow -\infty} (300 - 500x^{35} - 20x^{37} - 70x^2)$$

(4 pts)

$$= \lim_{x \rightarrow -\infty} (-20x^{37}) = +\infty$$

$$5. \quad \lim_{t \rightarrow 1} \frac{t^2 - 4t + 3}{t^3 + t - 2}$$

Note that as  $t \rightarrow 1$ , the given limit is of (6 pts) the form  $\frac{0}{0}$ . So  $(t-1)$  is a factor in both the numerator and the denominator. For the numerator, in this case, we may use synthetic division to get a factorization for  $t^3 + t - 2$  as follows:

$$= \lim_{t \rightarrow 1} \frac{(t-1)(t-3)}{(t-1)(t^2+t+2)}$$

$$= \lim_{t \rightarrow 1} \frac{t-3}{t^2+t+2}$$

$$= \frac{-2}{4}$$

$$= -\frac{1}{2}$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$t^3 + t - 2 = (t-1)(t^2 + t + 2)$

6. Let  $f(t) = \frac{1-t}{\sqrt{t^2-1}}$ . Using the **concept of limit**, find

(6 pts)

(a) all horizontal asymptotes (if any)

$$\begin{aligned} \lim_{t \rightarrow +\infty} \frac{1-t}{\sqrt{t^2-1}} &= \lim_{t \rightarrow +\infty} \frac{\frac{1}{|t|} - \frac{t}{|t|}}{\sqrt{\frac{t^2}{t^2} - \frac{1}{t^2}}} \\ &= \lim_{t \rightarrow +\infty} \frac{\frac{1}{t} - \frac{t}{t}}{\sqrt{1 - \frac{1}{t^2}}} = \lim_{t \rightarrow +\infty} \frac{\frac{1}{t} - 1}{\sqrt{1 - \frac{1}{t^2}}} = -1 \end{aligned}$$

Also,

$$\lim_{t \rightarrow -\infty} \frac{1-t}{\sqrt{t^2-1}} = \lim_{t \rightarrow -\infty} \frac{\frac{1}{|t|} - \frac{t}{|t|}}{\sqrt{\frac{t^2}{t^2} - \frac{1}{t^2}}} = \lim_{t \rightarrow -\infty} \frac{-\frac{1}{t} + 1}{\sqrt{1 - \frac{1}{t^2}}} = 1$$

Hence,  $y=1$  and  $y=-1$  are horizontal asymptotes.

(b) all vertical asymptotes (if any)

[Note that the denominator is zero when  $t=1, t=-1$ ]

$$\begin{aligned} \lim_{t \rightarrow 1} \frac{1-t}{\sqrt{t^2-1}} &= \lim_{t \rightarrow 1} \frac{1-t}{\sqrt{t^2-1}} \cdot \frac{\sqrt{t^2-1}}{\sqrt{t^2-1}} \\ &= \lim_{t \rightarrow 1} \frac{(1-t)\sqrt{t^2-1}}{t^2-1} \\ &= \lim_{t \rightarrow 1} \frac{-(t-1)\sqrt{t^2-1}}{(t-1)(t+1)} \\ &= \lim_{t \rightarrow 1} \frac{-\sqrt{t^2-1}}{t+1} = \frac{0}{2} = 0 \Rightarrow \text{No V.A. at } t=1. \end{aligned}$$

$$\lim_{t \rightarrow -1} \frac{1-t}{\sqrt{t^2-1}} \text{ DNE, since } \lim_{t \rightarrow -1} (1-t) = 2 \text{ but } \lim_{t \rightarrow -1} \sqrt{t^2-1} = 0.$$

$\therefore t=-1$  is a vertical asymptote.

7. Use the  $(\epsilon, \delta)$  definition of limit to show that  $\lim_{x \rightarrow -1} (3x+7) = 4$ .

(5 pts)

We need to show that for any  $\epsilon > 0$ ,  $\exists \delta > 0$  such that

$$|(3x+7)-4| < \epsilon \quad \text{if} \quad 0 < |x-(-1)| < \delta$$

$$\text{i.e.} \quad |3x+3| < \epsilon \quad \text{if} \quad 0 < |x+1| < \delta.$$

Now,  $|3x+7)-4| = |3x+3| = 3|x+1| < 3\delta$  (when  $|x+1| < \delta$ )

Letting  $\epsilon = 3\delta$ , so that  $\delta = \frac{\epsilon}{3}$ , we have

$$\text{for } \epsilon > 0, \exists \delta > 0 \text{ such that } |(3x+7)-4| < \epsilon \text{ if } 0 < |x+1| < \delta.$$

This shows that  $\lim_{x \rightarrow -1} (3x+7) = 4$ .

8. Find all the points where the function  $f(x) = \begin{cases} 8 & \text{if } x=1 \\ \frac{4x^2-4}{x-1} & \text{if } 1 < x < 3 \\ 12 & \text{if } x=3. \end{cases}$

is not continuous in the interval  $[1, 3]$ .

(6 pts)

We need to check the continuity of  $f$  on  $(1, 3)$  and on the right of 1 and on the left of 3, and also we check the changing point since  $f$  is a piecewise function. Note that  $f$  is continuous when it is a polynomial.

Now,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{4x^2-4}{x-1} = \lim_{x \rightarrow 1^+} \frac{4(x^2-1)}{x-1} = \lim_{x \rightarrow 1^+} \frac{4(x-1)(x+1)}{x-1} = 8 = f(1)$$

$\Rightarrow f$  is continuous at the right of  $x=1$ .

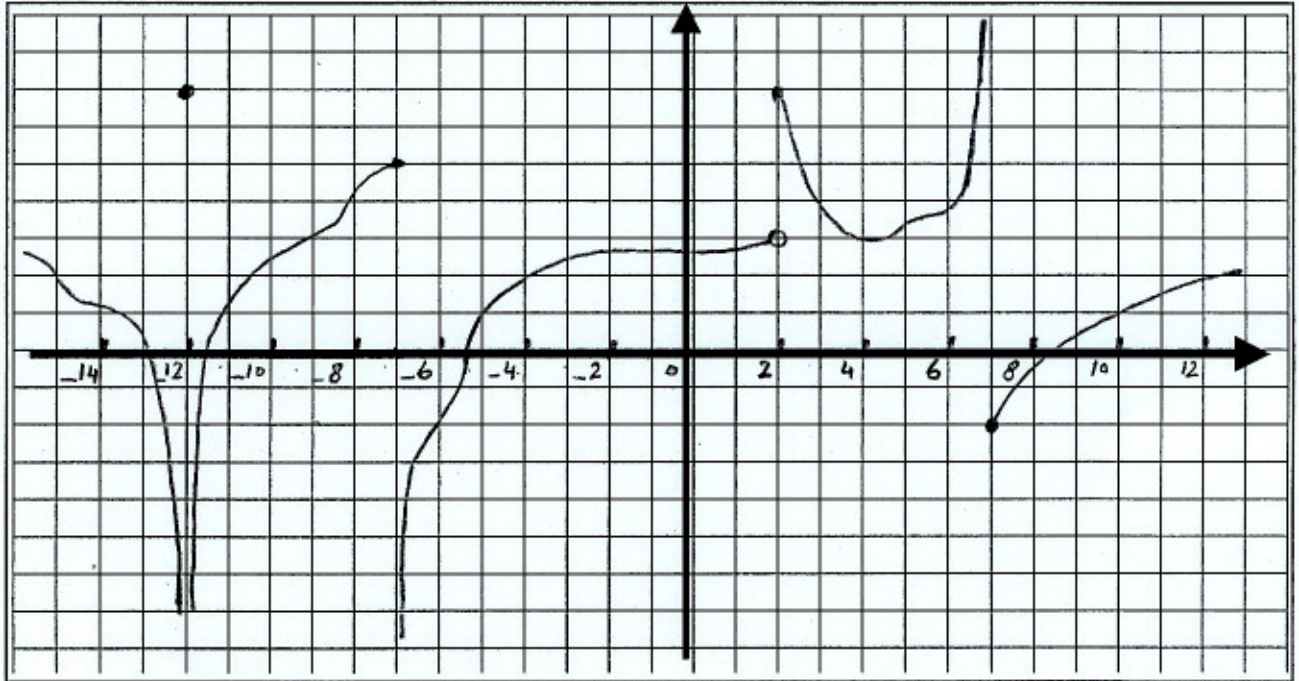
Next

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \frac{4x^2-4}{x-1} = 16 \neq 12 \quad \text{i.e.} \quad \lim_{x \rightarrow 3^-} f(x) \neq f(3)$$

$\therefore$  the given function has discontinuity at  $x=3$ .

9. Looking at the given graph of  $f(x)$ , answer the following questions:

(6 pts)



a.  $f(x)$  is **Not Left Continuous** at the point(s)  $x = \underline{-12, 2, 7}$

b.  $f(x)$  is **Not Right Continuous** at the point(s)  $x = \underline{-12, -7}$

c.  $\lim_{x \rightarrow 2^-} f(x) = \underline{3}$

d.  $\lim_{x \rightarrow 7^+} f(x) = \underline{-2}$

e.  $\lim_{x \rightarrow -12} f(x) = \underline{-\infty}$

f.  $f(x)$  is **Not Continuous** at the point(s)  $x = \underline{-12, -7, 2, 7}$