Department of Mathematical Sciences, KFUPM

Time: 50 Min. **MATH 101-051** **Points:_____/50**

ID________ **Section: 4, 8, 12, 17** **Serial #:_____**

Note: Show all work. Solution **No Calculator is allowed.**

Evaluate the limits in Questions 1-5

1. $\lim_{x\to 0}\frac{\sqrt{5+x}-\sqrt{5}}{x}=$ (6 pts)

$$= \lim_{x\to 0} \frac{\sqrt{5+x}-\sqrt{5}}{x}\cdot\frac{\sqrt{5+x}+\sqrt{5}}{\sqrt{5+x}+\sqrt{5}}$$

$$= \lim_{x\to 0} \frac{\cancel{5}+x-\cancel{5}}{x(\sqrt{5+x}+\sqrt{5})} = \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{5+x}+\sqrt{5})}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{5+x}+\sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}}$$

2. $\lim_{x\to\infty}\frac{\sqrt[3]{27x^6-809x+1}}{10x^2-1}$ (6 pts)

$$= \lim_{x\to\infty}\frac{\sqrt[3]{\frac{27x^6}{x^6}-\frac{809x}{x^6}+\frac{1}{x^6}}}{\frac{10x^2}{x^2}-\frac{1}{x^2}}$$

$$\left[\sqrt[3]{x^6}=x^2\right]$$

$$= \lim_{x\to\infty}\frac{\sqrt[3]{27-\frac{809}{x^5}+\frac{1}{x^6}}}{10-\frac{1}{x^2}}$$

$$= \frac{3}{10}$$

3. $\lim\limits_{x\to 4^-}\dfrac{2x-8}{|4-x|}=$ (5 pts)

$$= \lim_{x\to 4^-} \frac{2(x-4)}{|4-x|}$$

$$= \lim_{x\to 4^-} \frac{2(x-4)}{4-x} = \lim_{x\to 4^-} \frac{-2(4-x)}{4-x}$$

$$= -2$$

4. $\lim\limits_{t\to -\infty}\left(300+200t-50t^4-70t^2\right)$ (4 pts)

$$= \lim_{t\to -\infty} \left(-50t^4\right) = -\infty$$

5. $\lim\limits_{t\to 1}\dfrac{t^3+t-2}{t^2+t-2}$ (6 pts)

Note that as $t\to 1$, the given limit is of the form $\frac{0}{0}$. So $(t-1)$ is a common factor in both the numerator and the denominator. For the numerator, in this case, we may use synthetic division to obtain a factorization for t^3+t-2 as follows:

$$= \lim_{t\to 1} \frac{\cancel{(t-1)}(t^2+t+2)}{\cancel{(t-1)}(t+2)}$$

$$= \lim_{t\to 1} \frac{t^2+t+2}{t+2}$$

$$= \frac{4}{3}$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$$t^3+t-2 = (t-1)(t^2+t+2)$$

6. Let $f(t)=\dfrac{4-t}{\sqrt{t^2-16}}$. Using the **concept of limit,** find (6 pts)

(a) all **horizontal asymptotes** (if any)

$$\lim_{t\to+\infty}\frac{4-t}{\sqrt{t^2-16}} = \lim_{t\to+\infty}\frac{\frac{4}{|t|}-\frac{t}{|t|}}{\sqrt{\frac{t^2}{t^2}-\frac{16}{t^2}}}$$

$$= \lim_{t\to+\infty}\frac{\frac{4}{t}-\frac{t}{t}}{\sqrt{1-\frac{16}{t^2}}} = \lim_{t\to+\infty}\frac{\frac{4}{t}-1}{\sqrt{1-\frac{16}{t^2}}} = -1$$

Also,

$$\lim_{t\to-\infty}\frac{4-t}{\sqrt{t^2-16}} = \lim_{t\to-\infty}\frac{\frac{4}{|t|}-\frac{t}{|t|}}{\sqrt{\frac{t^2}{t^2}-\frac{16}{t^2}}} = \lim_{t\to-\infty}\frac{-\frac{4}{t}+1}{\sqrt{1-\frac{16}{t^2}}} = 1$$

Hence, $y=1$ and $y=-1$ are horizontal asymptotes.

(b) all **vertical asymptotes** (if any)

[Note that the denominator is zero when $t=4$, $t=-4$.]

$$\lim_{t\to 4}\frac{4-t}{\sqrt{t^2-16}} = \lim_{t\to 4}\frac{4-t}{\sqrt{t^2-16}}\cdot\frac{\sqrt{t^2-16}}{\sqrt{t^2-16}}$$

$$= \lim_{t\to 4}\frac{(4-t)\sqrt{t^2-16}}{t^2-16} = \lim_{t\to 4}\frac{-\cancel{(t-4)}\sqrt{t^2-16}}{\cancel{(t-4)}(t+4)}$$

$$= \lim_{t\to 4}\frac{-\sqrt{t^2-16}}{t+4} = \frac{0}{8} = 0$$

So, there is No vertical asymptote at $t=4$.

$\lim_{t\to -4}\frac{4-t}{\sqrt{t^2-16}}$ DNE, since $\lim_{t\to-4}(4-t)=8$ but $\lim_{t\to-4}\sqrt{t^2-16}=0$

$\therefore$ $t=-4$ is a vertical asymptote.

7. **Use the (ε, δ) definition of limit** to show that $\lim_{x \to -2}(3x+7) = 1$. (5 pts)

We need to show that for any $\epsilon > 0$, $\exists\, \delta > 0$ such that

$$|(3x+7)-1| < \epsilon \quad \text{if } 0 < |x-(-2)| < \delta.$$

i.e. $|3x+6| < \epsilon$ if $0 < |x+2| < \delta$.

Now, $|(3x+7)-1| = |3x+6| = 3|x+2| < 3\delta$ (when $|x+2| < \delta$)

Letting $\epsilon = 3\delta$ so that $\delta = \epsilon/3$, we have

for $\epsilon > 0$, $\exists\, \delta > 0$ such that $|(3x+7)-1| < \epsilon$ if $0 < |x+2| < \delta$.

This shows that $\lim_{x \to -2} (3x+7) = 1$.

8. **Find all the points** where the function $f(x) = \begin{cases} -5 & \text{if} \quad x = -1 \\ \dfrac{2x^2-2}{x+1} & \text{if} \quad -1 < x < 3 \\ 4 & \text{if} \quad x = 3. \end{cases}$

is **not continuous** in the interval $[-1, 3]$. (6 pts)

We need to check the continuity of f on $(-1, 3)$ and on the right of -1 and on the left of 3

and since f is a piecewise function, we need to check the changing points.

otherwise, f is continuous when it is a polynomial.

Now,

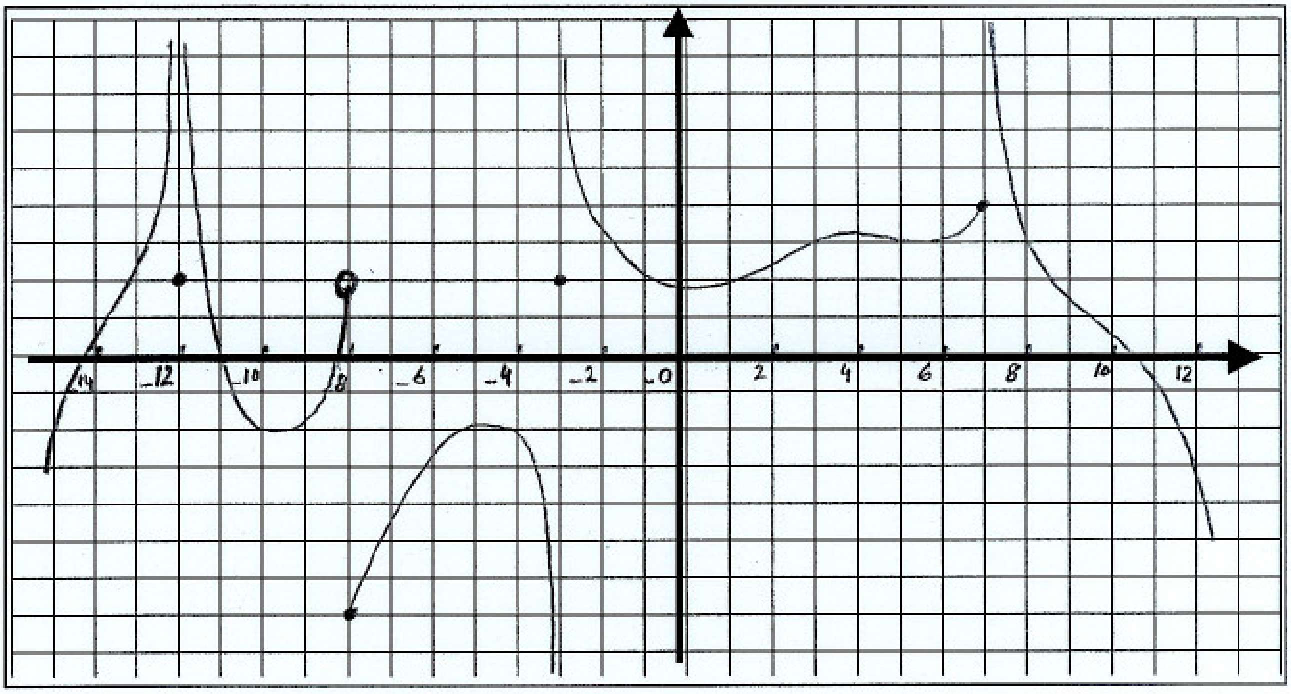
$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{2x^2-2}{x+1} = \lim_{x \to -1^+} \frac{2(x^2-1)}{x+1} = \lim_{x \to -1^+} \frac{2(x-1)\cancel{(x+1)}}{\cancel{x+1}} = -4 \neq -5$$

$\Rightarrow \lim_{x \to -1^+} f(x) \neq f(-1) \Rightarrow f$ is not continuous at $x = -1$

Next, Consider $x = 3 \Rightarrow f(3) = 4$ and $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{2x^2-2}{x+1} = 4 = f(3)$

$\therefore$ the given function has discontinuity only at $x = -1$.

9. **Looking at the given graph** of $f(x)$, **answer** the following questions: (6 pts)



a. $f(x)$ is **Not Left Continuous** at the point(s) $x =$ $-12, -8, -3$

b. $f(x)$ is **Not Right Continuous** at the point(s) $x =$ $-12, -3, 7$

c. $\lim_{x \to -8^-} f(x) =$ 2

d. $\lim_{x \to -3^+} f(x) =$ $+\infty$

e. $\lim_{x \to -12} f(x) =$ $+\infty$

f. $f(x)$ is **Not Continuous** at the point(s) $x =$ $-12, -8, -3, 7$