Section # __________ **Quiz 4 (MATH 102)** Sr # __________

Q. 1. Find the local quadratic approximation of $e^{2(x-1)}$ about $x = 1$.

$$P_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$$

$f(x) = e^{2(x-1)}$

$f(1) = 1$

$f'(x) = 2e^{2(x-1)}$

$f'(1) = 2$

$f''(x) = 4e^{2(x-1)}$

$f''(1) = 4$

Ans

$$\boxed{P_2(x) = 1 + 2(x-1) + \frac{4}{2!}(x-1)^2}$$

Q2. If we want to approximate $\sqrt{9.1}$ by using Taylor Series of a function $f(x)$ about $x = a$, then (fill in the blanks)

$f(x) =$ $\sqrt{x}$ and $a =$ 3

Q4. Write the general term of the sequence: $\frac{2}{5}, \frac{4}{8}, \frac{8}{11}, \ldots\ldots$

$a_1 = \frac{2}{5} = 2^1/(3(1)+2)$

$a_2 = \frac{4}{8} = 2^2/(3(2)+2)$

$a_3 = \frac{8}{11} = 2^3/(3(3)+2)$

$\vdots$

$a_n = \boxed{\dfrac{2^n}{3n+2}}$

Q4. Evaluate: $\lim\limits_{x\to\infty}\left(\dfrac{n+2}{2n-1}\right)^{3n}$.

① Here $y = \left(\dfrac{x+2}{2x-1}\right)^{3x}$ (1^∞ form)

② $\ln y = \ln\left(\dfrac{x+2}{2x-1}\right)^{3x}$

$\ln y = 3x\ln\left(\dfrac{x+2}{2x-1}\right)$ ($\infty \cdot 0$ form)

$\Rightarrow \ln y = 3\ln\left(\dfrac{x+2}{2x-1}\right)/(1/x)$ ($0/0$ form)

③ $\lim\limits_{x\to\infty}\ln y = 3\lim\limits_{x\to\infty}\ln\left(\dfrac{x+2}{2x-1}\right)/(1/x)$

$= 3\lim\limits_{x\to\infty}\dfrac{2x-1}{x+2}\cdot\dfrac{(2x-1)-(x+2)2}{(2x-1)^2}\Big/\left(\dfrac{-1}{x^2}\right)$

$= 3\lim\limits_{x\to\infty}\dfrac{2x-1}{x+2}\cdot\dfrac{+5x^2}{(2x-1)^2} = 3\left(\dfrac{10}{4}\right) = \dfrac{15}{2}$

④ $\ln\left(\lim\limits_{x\to\infty} y\right) = \dfrac{15}{2} \Rightarrow \lim\limits_{x\to\infty} y = e^{15/2}$

⑤ Ans $\boxed{\lim\limits_{n\to\infty}\left(\dfrac{n+2}{2n-1}\right)^{3n} = e^{15/2}}$

Section # __________ **Quiz 4 (MATH 102)** Sr # __________

Q. 1. Find the local quadratic approximation of $\cos x$ about $\frac{\pi}{4}$.

$$p_2(x) = f\left(\frac{\pi}{4}\right) + f'\left(\frac{\pi}{4}\right)\left(x-\frac{\pi}{4}\right) + \frac{f''\left(\frac{\pi}{4}\right)}{2!}\left(x-\frac{\pi}{4}\right)^2$$

$f(x) = \cos x$

$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$f'(x) = -\sin x$

$f'\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$f''(x) = -\cos x$

$f''\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Ans

$$\boxed{p_2(x) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}}\left(x-\frac{\pi}{4}\right)^2}$$

Q2. If we want to approximate $\sqrt{4.2}$ by using Taylor Series of a function $f(x)$ about $x = a$, then (fill in the blanks)

$f(x) =$ $\sqrt{x}$ and $a =$ 2

Q4. Write the general term of the sequence: $\frac{4}{3}, \frac{8}{7}, \frac{12}{11}, \ldots\ldots$

$a_1 = 4/3 = 4\cdot1/(4(1)-1)$

$a_2 = 8/7 = 4\cdot2/(4(2)-1)$

$a_3 = 12/11 = 4\cdot3/(4(3)-1)$

$\vdots$

$a_n = \boxed{4\cdot n/(4n-1)}$

Q4. Evaluate: $\lim\limits_{x\to\infty}\left(\frac{3n+1}{n-2}\right)^{2n}$.

(1) Here $y = \left(\frac{3x+1}{x-2}\right)^{2x}$ (1^∞ form)

(2) $\ln y = \ln\left(\frac{3x+1}{x-2}\right)^{2x}$

$\ln y = 2x \ln\left(\frac{3x+1}{x-2}\right)$ ($\infty\cdot 0$ form)

$\Rightarrow \ln y = 2 \ln\left(\frac{3x+1}{x-2}\right)/(1/x)$ ($0/0$ form)

(3) $\lim\limits_{x\to\infty} \ln y = 2 \lim\limits_{x\to\infty} \ln\left(\frac{3x+1}{x-2}\right)/(1/x)$

$= 3 \lim\limits_{x\to\infty} \frac{x-2}{3x+1} \cdot \frac{(x-2)\cdot 3 - (3x+1)}{(x-2)^2} \Big/ \left(\frac{-1}{x^2}\right)$

$= 3 \lim\limits_{x\to\infty} \frac{x-2}{3x+1} \cdot \frac{5x^2}{(x-2)^2} = 3\left(\frac{5}{3}\right) = 5$

(4) $\ln\left(\lim\limits_{x\to\infty} y\right) = 5 \Rightarrow \lim\limits_{x\to\infty} y = e^5$

(5) Ans $\boxed{\lim\limits_{n\to\infty}\left(\frac{3n+1}{n-2}\right)^{2n} = e^5}$