Math 311 Quiz # 2

Name: Solution I.D. # ____________

1. Show that the limit of a function, if it exists, is unique.

Assume that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} f(x) = M$ where $L \neq M$.

set $|L-M| = \epsilon$, so that $\epsilon > 0$ — — — - (*)

For $\epsilon/2 > 0$, $\exists \delta_1 > 0$ such that $0 < |x-a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon/2$,

and also $\exists \delta_2 > 0$ such that $0 < |x-a| < \delta_2 \Rightarrow |f(x) - M| < \epsilon/2$.

let $\delta = \min\{\delta_1, \delta_2\}$. Then $\delta > 0$ and $0 < |x-a| < \delta \Rightarrow \begin{cases} |f(x) - L| < \epsilon/2 \\ |f(x) - M| < \epsilon/2 \end{cases}$.

Thus, $|L - M| = |L - f(x) + f(x) - M| \leq |f(x) - L| + |f(x) - M| < \epsilon/2 + \epsilon/2 = \epsilon$

which is a Contradiction to (*). Hence $L = M$ and the limit is unique.

2. Use the Sandwich theorem to find $\lim_{x\to\infty} \frac{1}{x}\sin x$.

Recall, $-1 \leq \sin x \leq 1$, then

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

Now, $\lim_{x\to\infty}\left(-\frac{1}{x}\right) = 0$, and $\lim_{x\to\infty} \frac{1}{x} = 0$. Hence

by the Sandwich theorem we conclude that $\lim_{x\to\infty} \frac{\sin x}{x} = 0$.

3. Determine whether the function g is continuous at $x = 5$ or not, where g is given by

$$g(x) = \begin{cases} \sqrt{\dfrac{x^2 - 3x - 10}{x-5}}, & 5 < x < 10 \\ 7x & -1 < x \leq 5 \end{cases}$$

$$\lim_{x\to 5^+} g(x) = \lim_{x\to 5^+} \sqrt{\frac{x^2-3x-10}{x-5}} = \lim_{x\to 5^+} \sqrt{\frac{\cancel{(x-5)}(x+2)}{\cancel{x-5}}} = \lim_{x\to 5^+} \sqrt{x+2} = \sqrt{7}$$

$$\lim_{x\to 5^-} g(x) = \lim_{x\to 5^-} 7x = 35$$

Hence $\lim_{x\to 5} g(x)$ does not exist and so the function is not continuous at $x = 5$.