Math 311 Quiz # 1

Name: Solution I.D. # ____________

1. Determine which of the following forms a field:

i) The set of all integers

Not a field: the multiplicative inverse is not satisfied.

ii) The set of all rational numbers

IS a field

iii) The set of all 2×2 matrices over the real numbers.

Not a field; the multiplicative inverse is not satisfied.

For example: $A=\begin{bmatrix}2 & 1\\ 6 & 3\end{bmatrix}$ does not have an inverse.

2. Define an operation $*$ on the set of real numbers as follows: $a*b=a+b+ab$.

i) Prove or disprove that the operation $*$ is associative.

$$(a*b)*c = (a+b+ab)*c = (a+b+ab)+c+(a+b+ab)c$$

$$= a+b+ab+c+ac+bc+abc$$

$$= a+b+c+ab+ac+bc+abc$$

Similarly, $a*(b*c) = a*(b+c+bc) = a+(b+c+bc)+a(b+c+bc) = \dots = a+b+c+ab+ac+bc+abc$

$\therefore *$ is associative.

ii) Find the identity element e for this operation on R.

the identity element is 0, since $a*0 = a+0+a\cdot0 = a$

$0*a = \dots = a, \ \forall a\in R$.

iii) What is the inverse of -3 under this operation?

Let b be the inverse of -3, then

$-3*b=0$

$\Rightarrow -3*b = -3+b+(-3)b = 0$

$\Rightarrow b=-\frac{3}{2}$

i.e. the inverse of -3 is $-\frac{3}{2}$.

3. **Prove that for any real numbers x and y, $x(-y) = -(xy)$.**

Consider $\quad x(-y) + xy = x(-y+y)$

$$= x \cdot 0$$

$$= 0$$

$$\Rightarrow \quad x(-y) = -(xy)$$

4. **Solve the inequality: $\frac{1}{x} < 1$.** $\quad , \quad x \neq 0$

We consider two cases ; $\quad x > 0 \quad$ or $\quad x < 0$

$\underline{x > 0}$:

$$\frac{1}{x} < 1$$

$$\Rightarrow \quad x > 1$$

$\therefore S_1 = \{x \in \mathbb{R} \mid x > 1 \text{ and } x > 0\}$

$$= (1, \infty)$$

$\underline{x < 0}$:

$$\frac{1}{x} < 1$$

$$\Rightarrow \quad 1 > x$$

$S_2 = \{x \in \mathbb{R} : x < 1 \text{ and } x < 0\}$

$$= (-\infty, 0)$$

$\therefore$ the solution set is $\quad S = S_1 \cup S_2$

$$= (-\infty, 0) \cup (1, \infty).$$