

1. Use the $\epsilon - \delta$ definition to prove that $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$.
2. Find the limit of each of the following:

(a) $\lim_{x \rightarrow 5} \frac{x^2 - 25}{|x - 5|}$

(b) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$

(c) $\lim_{x \rightarrow 2} \frac{x^2 - x + 6}{x - 2}$

(d) $\lim_{x \rightarrow 0^-} \left[\frac{1}{x} - \frac{1}{|x|} \right]$

(e) $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$

(f) $\lim_{x \rightarrow 0} \frac{\sin^3 x}{x^3}$

(g) $\lim_{x \rightarrow 0^+} (1 + x)^{1/x}$

(h) $\lim_{x \rightarrow 1} \sin^{-1} \left(\frac{1 - \sqrt{x}}{1 - x} \right)$

(i) $\lim_{x \rightarrow \infty} \frac{e^{3x}}{x^4}$

(j) $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$

(k) $\lim_{x \rightarrow 0^+} (\cos x)^{1/x^2}$

3. Find the derivative y' for each of the following:

(a) $y = \sqrt{e^{2x} - \csc^3 x}$

(b) $y = \frac{\sec^2 5x^2 + 1}{1 + \cos^{-1} x}$

(c) $xy = \cot(xy)$

(d) $yx + 1 = 3 \tan^{-1} y$

(e) $y = \sin(\tan \sqrt{\sin x})$

(f) $y = 10^{\sin x^2}$

4. If $x^4 + y^4 = 16$, show that $y'' = -48 \frac{x^2}{y^7}$.
5. Suppose f is a one-to-one differentiable function and its inverse f^{-1} also differentiable. Use implicit differentiation to show that

$$\frac{d}{dx} [f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}, \quad \text{where } f' \neq 0.$$

6. Use the definition of derivative to show that $\frac{d}{dx} [\log_b x] = \frac{1}{x \ln b}$, $x > 0$.
7. show that the equation $4x^3 - 6x^2 + 3x - 2 = 0$ has a real root between 1 and 2.
8. Show that the function $f(x) = |x - 3|$ is continuous everywhere.
9. Given

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ e^x & \text{if } x < 0. \end{cases}$$

Discuss the continuity of f at $x = 0$.

10. Find the horizontal and vertical asymptotes of the graph of $f(x) = \frac{\sqrt{2x^2 + 1}}{3x - 5}$.
11. Find the critical points of $f(x) = 4x^{3/5} - x^{8/5}$.
12. Find the absolute max and absolute min of $f(x) = x^4 - 2x^2 + 3$ on $[-2, 3]$.
13. Sketch the graph of $\frac{2x - 5}{x + 3}$.
14. State Rolle's theorem and verify that the function $f(x) = \sin 2\pi x$ satisfies the hypotheses of Rolle's theorem on the interval $[-1, 1]$. Then find a number c that satisfies its conclusion on this interval.
15. Is it true that the equation $y = y''' + 5y' - 6$ is satisfied by $y = x$?
16. Is it true that the inverse function of $y = \sin x$ is $y = \frac{1}{\sin x}$?
17. Is it true that the function $y = \ln x$ is differentiable everywhere?
18. Is it true that if $k(x) = f(g(x))$, then $\frac{d^2 k}{dx^2} = f'(g) \cdot g'' + f''(g) \cdot (g')^2$?
19. What is the error in the following steps:
- $$\lim_{x \rightarrow 0} \frac{\sin x}{x^2} = \lim_{x \rightarrow 0} \frac{\cos x}{2x} = \lim_{x \rightarrow 0} \frac{-\sin x}{2} = 0$$
- and determine the correct value of this limit.
20. Use local linear approximation to approximate $\sin 29^\circ$.