# Math 101 Quiz # 2

Name: ______________________________ I.D. # ____________ Section # ____ Serial #

**1.** Find each of the following limits:

i) $\lim_{x\to\infty} \frac{x^2-5}{x^2+5} = 1$

ii) $\lim_{x\to-\infty} \frac{x^2-5}{x^2+5} = 1$

**2.** Find the horizontal and vertical asymptotes of the graph of the function

$$f(x) = \frac{\sqrt{2x^2+1}}{7x-1}$$

$7x-1=0 \Rightarrow \boxed{x=\frac{1}{7}}$ is a V. A.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{7x-1} = \lim_{x\to\infty} \frac{\sqrt{\frac{2x^2}{x^2}+\frac{1}{x^2}}}{\frac{7x}{|x|}-\frac{1}{|x|}} = \lim_{x\to\infty} \frac{\sqrt{2+\frac{1}{x^2}}}{7-\frac{1}{x}} = \frac{\sqrt{}}{7}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{7x-1} = \ldots\ldots = \lim_{x\to-\infty} \frac{\sqrt{2+\frac{1}{x^2}}}{-7+\frac{1}{x}} = -\frac{\sqrt{2}}{7}$$

$\therefore$ The graph of $f(x)$ has H. A. $\boxed{y=\frac{\sqrt{2}}{7}}$ and $\boxed{y=-\frac{\sqrt{2}}{7}}$

**3.** Use the definition to prove that $\lim_{x\to-2} x^2-1=3$

We show that Given $\epsilon>0$, $\exists\ \delta>0$ s.t. $|x-(-2)|<\delta \Rightarrow |(x^2-1)-3|<$

i.e. $|x+2|<\delta \Rightarrow |x^2-4|<$

Now, if $|x+2|<\delta$, we have

$$|x^2-4| = |(x-2)(x+2)| = |(x+2)(x+2-4)| = |x+2|\,|x+2-4|$$
$$\le |x+2|(|x+2|+4)$$
$$< \delta(\delta+4),$$
$$= \delta^2+4\delta$$
$$< \delta+4\delta$$
$$= 5\delta$$

...nce, taking $\epsilon=5\delta$ so that:

$\delta=\min(\frac{\epsilon}{5},1)$, we get

$|x^2-4|<\epsilon$ whenever $|x+2|<\delta$ as desired.