Math 260 Quiz # 5

Name: ______________________________ I.D. # ____________ Section # ____ Serial #

1. Show that e^x, e^{2x} and e^{3x} form a linearly independent set of solutions for the DE $y''' - 6y'' + 11y' - 6y = 0$. Then write the general solution of this equation.

Consider $y = e^x$. Then $y' = y'' = y''' = e^x$.

Substitute in the given DE, we get

$$y''' - 6y'' + 11y' - 6y = e^x - 6e^x + 11e^x - 6e^x = 0.$$

Hence $y = e^x$ is a solution. Similarly we can verify that e^{2x} and e^{3x} are solutions.

Now, to show that these solutions are linearly independent, we compute

$$W(e^x, e^{2x}, e^{3x}) = \begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix} = e^x e^{2x} e^{3x} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$$

$$= e^{6x}(2) = 2e^{6x} \neq 0$$

Hence e^x, e^{2x} and e^{3x} form a linearly independent set of solutions.

The general solution is $y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x}$

2. Find the general solution of the following DE: $y'' + 2y' - 15y = 0$

The characteristic equation is $\lambda^2 + 2\lambda - 15 = 0$

$$\Rightarrow (\lambda - 3)(\lambda + 5) = 0 \Rightarrow \lambda = 3, -5$$

The general solution is $y(x) = c_1 e^{3x} + c_2 e^{-5x}$

Math 260 Quiz # 5b

Name: ______________________________ I.D. # ____________ Section # ____ Serial #

1. Show that e^x, e^{2x} and e^{3x} form a linearly independent set of solutions for the DE $y''' - 6y'' + 11y' - 6y = 0$. Then write the general solution of this equation.

See Quiz # 5

2. Find the general solution of the following DE: $y'' + 5y' = 0$

The characteristic equation is $\lambda^2 + 5\lambda = 0$

$\Rightarrow \lambda(\lambda + 5) = 0 \Rightarrow \lambda = 0, -5$

The general solution is $y(x) = c_1 + c_2 e^{-5x}$

Math 260 Quiz # 5c

Name: ______________________________ I.D. # ____________ Section # ____ Serial #

1. Show that e^x, e^{2x} and e^{3x} form a linearly independent set of solutions for the DE $y''' - 6y'' + 11y' - 6y = 0$. Then write the general solution of this equation.

See Quiz # 5

2. Find the general solution of the following DE: $2y'' + 3y' = 0$

The characteristic equation is $2\lambda^2 + 3\lambda = 0$

$\Rightarrow \lambda(2\lambda + 3) = 0 \Rightarrow \lambda = 0, -\frac{3}{2}$

The general solution is $y(x) = c_1 + c_2 e^{-\frac{3}{2}x}$

Math 260 Quiz # 5d

Name: ______________________________ I.D. # ____________ Section # ____ Serial #

1. Show that e^{x}, e^{2x} and e^{3x} form a linearly independent set of solutions for the DE $y''' - 6y'' + 11y' - 6y = 0$. Then write the general solution of this equation.

See Quiz # 5

2. Find the general solution of the following DE: $2y'' - y' - y = 0$

The characteristic equation is $2\lambda^2 - \lambda - 1 = 0$

$\Rightarrow (\lambda - 1)(2\lambda + 1) = 0 \quad \Rightarrow \lambda = 1, -\frac{1}{2}$

The general solution is $y(x) = c_1 e^{x} + c_2 e^{-\frac{x}{2}}$