Math 260 Quiz # 3

Name: ______________________________ I.D. # ____________ Section # ____

Solve the DE $\quad y' = \dfrac{e^x \cos y - 2xy}{e^x \sin y + x^2}$

$$\frac{dy}{dx} = \frac{e^x \cos y - 2xy}{e^x \sin y + x^2}$$

$$\underset{M}{\left(e^x \cos y - 2xy\right)} dx + \underset{N}{\left(-e^x \sin y - x^2\right)} dy = 0$$

$$\frac{\partial M}{\partial y} = -e^x \sin y - 2x \,, \quad \frac{\partial N}{\partial x} = -e^x \sin y - 2x$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{Exact.}$$

$$\therefore \frac{\partial f}{\partial x} = M = e^x \cos y - 2xy \,, \quad \frac{\partial f}{\partial y} = N = -e^x \sin y - x^2$$

$$f(x,y) = \int \left(e^x \cos y - 2xy\right) dx = e^x \cos y - x^2 y + g(y)$$

$$\frac{\partial f}{\partial y} = -e^x \sin y - x^2 + g'(y) = N$$

$$\Rightarrow -e^x \sin y - x^2 + g'(y) = -e^x \sin y - x^2$$

$$\Rightarrow g'(y) = 0$$

$$\Rightarrow g(y) = C_1$$

$$\therefore f(x,y) = e^x \cos y - x^2 y + C_1$$

i.e. The solution is $\quad e^x \cos y - x^2 y = C$