Math 260 Quiz # 6

Name: ______________________ I.D. # __________ Section # ____ Serial #

Consider the DE $y'' - 2y' + 5y = e^x \cos 2x$

i) Find the solution of the associated homogeneous DE of the given equation.

Solving $y'' - 2y' + 5y = 0$

$\lambda^2 - 2\lambda + 5 = 0$

$\Rightarrow \lambda = \frac{2 \pm \sqrt{4-20}}{2} \Rightarrow \lambda = 1 \pm 2i$.

$\therefore$ the solution is $y_c = c_1 e^x \cos 2x + c_2 e^x \sin 2x$

ii) Write the general solution for the given non-homogeneous equation.

We have $(D^2 - 2D + 5) y = e^x \cos 2x$. $\quad Ann(e^x \cos 2x) = D^2 - 2D + 5$

$(D^2 - 2D + 5)(D^2 - 2D + 5) y = 0$ ———— $(*)$

$(D^2 - 2D + 5)^2 y = 0$

Solving this equation: $(\lambda^2 - 2\lambda + 5)^2 = 0 \Rightarrow \lambda = 1 \pm 2i, \ 1 \pm 2i$

Solution of $(*)$ is $y = \underbrace{c_1 e^x \cos 2x + c_2 e^x \sin 2x}_{y_c} + \underbrace{c_3 x e^x \cos 2x + c_4 x e^x \sin 2x}_{y_p}$

Comparing, we get

$y_p = A x e^x \cos 2x + B x e^x \sin 2x = x e^x [A \cos 2x + B \sin 2x]$

$y_p' = x e^x [-2A \sin 2x + 2B \cos 2x] + (x e^x + e^x)[A \cos 2x + B \sin 2x]$

$y_p'' = x e^x [-4A \cos 2x - 4B \sin 2x] + (x e^x + e^x)[-2A \sin 2x + 2B \cos 2x]$
$\quad + (x e^x + e^x)[-2A \sin 2x + 2B \cos 2x] + (x e^x + 2e^x)[A \cos 2x + B \sin 2x]$

Substituting in the DE $y'' - 2y' + 5y = e^x \cos 2x$, we get

$-4A e^x \sin 2x + 4B e^x \cos 2x = e^x \cos 2x$ [check !!]

$\Rightarrow 4B = 1 \Rightarrow \boxed{B = \frac{1}{4}}, \ -4A = 0 \Rightarrow \boxed{A = 0}$

$\therefore y_p = \frac{1}{4} x e^x \sin 2x$

$y = y_c + y_p = c_1 e^x \cos 2x + c_2 e^x \sin 2x + \frac{1}{4} x e^x \sin 2x$

Math 260 Quiz # 6b

Name: ______________________ I.D. # ____________ Section # ____ Serial #

Consider the DE $y'' + y' = 2 - \sin x$

i) Find the solution of the associated homogeneous DE of the given equation.

Solving $y'' + y' = 0$

$\lambda^2 + \lambda = 0$

$\lambda(\lambda+1) = 0 \implies \lambda = 0, -1$

$\therefore$ the solution is $y_c = c_1 + c_2 e^{-x}$

ii) Write the general solution for the given non-homogeneous equation.

We need to solve $(D^2+D)y = 2 - \sin x$. $\text{Ann}(2-\sin x) = D(D^2+1)$

$D(D^2+1)(D^2+D)y = 0$ ——— (*)

$D^2(D^2+1)(D+1)y = 0$

Solving this equation: $\lambda^2(\lambda+1)(\lambda^2+1) = 0$

$\implies \lambda = 0, 0, -1, i, -i$

Solution of (*) is $y = \underbrace{c_1}_{} + \overbrace{c_2 x}^{} + \underbrace{c_3 e^{-x}}_{} + \overbrace{c_4 \cos x + c_5 \sin x}^{}$ —— (**)

Comparing (**) with y_c, we get y_c

$y_p = Ax + B\cos x + C\sin x$

$y_p' = A - B\sin x + C\cos x$

$y_p'' = -B\cos x - C\sin x$

Substituting in the DE: $y'' + y' = 2 - \sin x$

$-B\cos x - C\sin x + A - B\sin x + C\cos x = 2 - \sin x$

$A + (C-B)\cos x + (-B-C)\sin x = 2 - \sin x$

$\implies \left.\begin{array}{l} -B-C=-1 \\ B-C=0 \\ A=2 \end{array}\right\} \implies \left.\begin{array}{l} B=\frac{1}{2} \\ C=\frac{1}{2} \end{array}\right\} \implies y_p = 2x + \frac{1}{2}\cos x + \frac{1}{2}\sin x$

$\therefore y = y_c + y_p = c_1 + c_2 e^{-x} + 2x + \frac{1}{2}\cos x + \frac{1}{2}\sin x$