C

Math 260 Quiz # 4c

Name: _____ I.D. # _____ Section # ____ Serial #

1. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -\frac{1}{2}$, and $A = \begin{bmatrix} a-g & b-h & c-i \\ 4d & 4e & 4f \\ -g & -h & -i \end{bmatrix}$ then $\det(A) = \ldots$

$$\begin{vmatrix} a-g & b-h & c-i \\ 4d & 4e & 4f \\ -g & -h & -i \end{vmatrix} = -\begin{vmatrix} a-g & b-h & c-i \\ 4d & 4e & 4f \\ g & h & i \end{vmatrix} = -4\begin{vmatrix} a-g & b-h & c-i \\ d & e & f \\ g & h & i \end{vmatrix} = -4\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -4\left(-\tfrac{1}{2}\right)$$

$$= 2$$

2. If $B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{bmatrix}$, find $(B^T)^{-1}$, if it exists $\quad B^T = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 2 & 4 & -2 \end{bmatrix}$.

$$[B^T|I] = \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 4 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3]{R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 & 1 & 0 \\ 0 & -2 & -2 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & -2 & -2 & -2 & 0 & 1 \\ 0 & 5 & 1 & 1 & 1 & 0 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 5 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & -4 & -4 & 1 & \frac{5}{2} \end{array}\right] \xrightarrow{-3R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -2 & 0 & \frac{3}{2} \\ 0 & 1 & 1 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & 1 & -\frac{1}{4} & -\frac{5}{8} \end{array}\right]$$

$$\xrightarrow[-R_3+R_2]{3R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & \frac{1}{8} \\ 0 & 0 & 1 & 1 & -\frac{1}{4} & -\frac{5}{8} \end{array}\right] = \left[I \,\middle|\, (B^T)^{-1}\right]$$

$$\therefore \quad (B^T)^{-1} = \begin{bmatrix} 1 & -\frac{3}{4} & -\frac{3}{8} \\ 0 & \frac{1}{4} & \frac{1}{8} \\ 1 & -\frac{1}{4} & -\frac{5}{8} \end{bmatrix}$$