

Math 202 Quiz # 7

Name: _____ I.D. # _____ Section # _____

1. Find the eigenvalues of $A = \begin{bmatrix} 1 & 3 & 8 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 & 8 \\ 0 & 2-\lambda & 5 \\ 0 & 0 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(2-\lambda)(1-\lambda) = 0$$

$\lambda = 0, 1, 2$ the eigen values of A .

2. Find an eigenvector of A with respect to the largest eigenvalue.

The largest eigen value is $\lambda = 2$. So we solve $(A - 2I)X = 0$.

$$\left[\begin{array}{ccc|c} 1-2 & 3 & 8 & 0 \\ 0 & 2-2 & 5 & 0 \\ 0 & 0 & 0-2 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -1 & 3 & 8 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & -2 & 0 \end{array} \right] \xrightarrow{\substack{-R_1 \\ \frac{1}{5}R_2 \\ -\frac{1}{2}R_3}} \left[\begin{array}{ccc|c} 1 & -3 & -8 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\xrightarrow{\substack{-R_2+R_3 \\ 8R_2+R_1}} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow x_3 = 0, \quad x_1 = 3x_2$$

Take $x_2 = 1$, then $x_1 = 3$.

\therefore the required eigen vector is $E_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$.