Math 202 Quiz # 6

Name: Solution I.D. # ____________ Section # ____

1. Solve the following system:
$$\begin{aligned} 5x_1+7x_2+4x_3+x_4&=5\\ x_1+x_2-2x_3+3x_4&=4\\ 2x_1+3x_2+3x_3-x_4&=3 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 5 & 7 & 4 & 1 & 5\\ 1 & 1 & -2 & 3 & 4\\ 2 & 3 & 3 & -1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4\\ 5 & 7 & 4 & 1 & 5\\ 2 & 3 & 3 & -1 & 3 \end{array}\right]$$

$$\xrightarrow[-2R_1+R_3]{-5R_1+R_2} \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4\\ 0 & 2 & 14 & -14 & -15\\ 0 & 1 & 7 & -7 & -5 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4\\ 0 & 1 & 7 & -7 & -15/2\\ 0 & 1 & 7 & -7 & -5 \end{array}\right]$$

$$\xrightarrow{-R_2+R_3} \left[\begin{array}{cccc|c} 1 & 1 & -2 & 3 & 4\\ 0 & 1 & 7 & -7 & -15/2\\ 0 & 0 & 0 & 0 & 5/2 \end{array}\right]$$

$\Rightarrow$ The system has no solution.

2. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$. Find the inverse of A, if it exists.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 4 & 1 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[-4R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[2R_3+R_1]{R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & -1 & -6 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11 & 2 & 2 \\ 0 & 1 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 6 & -1 & -1 \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$$