Math 202 Quiz # 5

Name: ______________________ I.D. # __________ Section # ____

1. Find the radius of convergence and interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{3^n}{n!} x^{n+1}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{3^{n+1}}{(n+1)!}x^{n+2}}{\frac{3^n}{n!}x^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{3x}{n+1}\right| = |x|\lim_{n\to\infty}\left|\frac{3}{n+1}\right|$$

$$= 0$$

$\therefore$ The series is absolutely convergent on $(-\infty, \infty)$

So $R = \infty$

2. Write the first 4 terms of the series $(1-2x+x^2-2x^3\ldots)(x+3x^2+5x^3-7x^4+x^5\ldots)$

$$= x + 3x^2 + 5x^3 - 7x^4 + x^5 \ldots$$
$$-2x^2 - 6x^3 - 10x^4 + 14x^5 \ldots$$
$$+ x^3 + 3x^4 + 5x^5 \ldots$$
$$-2x^4 - 6x^5 - 10x^6 + \ldots$$

$$= x + x^2 + 0 - 6x^4 + 14x^5$$

3. Find the recurrence relation for the series solutions of the DE $y'' + 2xy' + 2y = 0$ about $x_0 = 0$.

$$y = \sum_{n=0}^{\infty} c_n x^n \quad , \quad y' = \sum_{n=1}^{\infty} n c_n x^{n-1} \quad , \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

Substituting in the DE,

$$y'' + 2xy' + 2y =$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + 2x \sum_{n=1}^{\infty} n c_n x^{n-1} + 2 \sum_{n=0}^{\infty} c_n x^n = 0$$

$$\underbrace{\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}}_{\text{Put } k=n-2} + \underbrace{\sum_{n=1}^{\infty} 2n c_n x^n}_{k=n} + \underbrace{\sum_{n=0}^{\infty} 2 c_n x^n}_{k=n} = 0$$

$$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k + \sum_{k=1}^{\infty} 2k c_k x^k + \sum_{k=0}^{\infty} 2 c_k x^k = 0$$

$$2c_2 + \sum_{k=1}^{\infty} (k+2)(k+1) c_{k+2} x^k + \qquad + 2c_0 + \sum_{k=1}^{\infty} 2 c_k x^k = 0$$

$$2c_2 + 2c_0 + \sum_{k=1}^{\infty} \left[(k+2)(k+1) c_{k+2} + 2k c_k + 2 c_k \right] x^k = 0$$

$$\therefore\ 2c_2 + 2c_0 = 0 \ \ \& \ \ (k+2)(k+1) c_{k+2} + 2(k+1) c_k = 0$$

$$\Rightarrow\ c_2 = -c_0 \ \ \& \ \ c_{k+2} = -\frac{2}{k+2} c_k \quad , \quad k = 1, 2, 3, \ldots$$