Math 202 Quiz # 7-B

Name: _____ I.D. # _____ Section # ____

1. Find the eigenvalues of $A = \begin{bmatrix} 1 & 3 & 8 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$|A-\lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 & 8 \\ 0 & 2-\lambda & 5 \\ 0 & 0 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow -\lambda(1-\lambda)(2-\lambda) = 0$$

$\lambda = 0, 1, 2$ the eigen values of A.

2. Find an eigenvector of A with respect to the smallest eigenvalue.

The smallest eigen value is $\lambda = 0$. So we solve $(A - 0 \cdot I)X = 0$

$$\left[\begin{array}{ccc|c} 1 & 3 & 8 & 0 \\ 0 & 2 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 8 & 0 \\ 0 & 1 & 5/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-3R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 5/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow x_2 = -\frac{5}{2} x_3$, $x_1 = -\frac{1}{2} x_3$

Take $x_3 = -2$, then $x_1 = 1$, $x_2 = 5$

$\therefore$ the required eigen vector is $E_0 = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$