Q3. (8 marks)

The following table is a partial probability distribution for some company's projected profits (X = profit in SR 1000s) for the first year of operation.

(The negative value denotes a loss).

x	−100	0	50	100	150	200
P(x)	0.1	0.2	0.3	0.25	0.1	

a. What is the proper value for P(X = 200)?
b. What is the probability that the company will be profitable?
c. What is the probability that the company will make at least SR 100,000?
d. What is the expected profit for the company?

(2) a. $P(X=200) = 1-\left[P(X=-100)+P(X=0)+P(X=50)+P(X=100) + P(X=150)\right]$

$= 0.05$

(2) b. $P(\text{Company is profitable}) = P(X \geq 50)$

$= 0.70$

(2) c. $P(X \geq 100{,}000) = 0.25 + 0.1 + 0.05 = 0.40$

(2) d. Expected Profit (in Thousands) $= (-100)(0.1) + (0)(0.2) + 50(0.3) + 100(0.25) + 150(0.1) + 200(0.05)$

$= -10 + 15 + 25 + 15 + 10$

(1) $\boxed{\text{Expected Profit} = \text{SR } 55{,}000}$

Q4. (6 Marks)

Small cars constitute 20% of vehicles on the road. The percentage of accidents involving small cars leading to a fatality (death) is 12%, and the percentage of accidents NOT involving small cars leading to a fatality is 5%.

a. What is the probability that an accident will <u>not</u> lead to a fatality?
b. Suppose that a reported accident have led to a fatality, what is the probability that a small car was involved?

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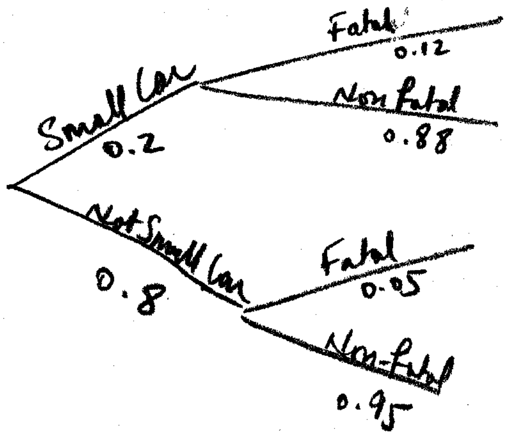

a. P(Non-fatal accident)

$= (0.2)(0.88) + (0.8)(0.95)$

$= \underline{0.176} + \underline{0.76}$

2 $= 0.936$

2 b. P(Small Car | Fatal Accident)

$$= \frac{P(\text{Small Car \& Fatal Accident})}{P(\text{Fatal Accident})} = \frac{(0.2)(0.12)}{0.064}$$

$$= \frac{0.024}{0.064} = 0.375$$

Q5. (6 Marks)

Historical data show that the average number of patient arrivals at the intensive care unit of General Hospital is 3 patients every 2 hours. Assume that the patient arrivals are distributed according to Poisson distribution. Determine the probability of 6 patients arriving in a five-hour period.

2 $\lambda = 3$ patients / 2 hrs
$= 1.5$ patients / hr.

3 $P(\#\text{ of patients is 6 in 5 hours}) = \dfrac{e^{-(1.5)5}(7.5)^6}{6!}$

1 $\longrightarrow$ $= 0.1367$

Q6. (9 Marks)

Suppose that we have a sample space S = {E_1, E_2, E_3, E_4, E_5, E_6, E_7} with the following probabilities:

$P(E_1) = 0.05$, $P(E_2) = P(E_3) = 0.20$, $P(E_4) = 0.25$
$P(E_5) = 0.15$, $P(E_6) = 0.10$, $P(E_7) = 0.05$

and let $A = \{E_1, E_4, E_6\}$ and $B = \{E_2, E_4, E_7\}$.

a. Find P(A) and $P(A \cap B)$.
b. Find $P(\bar{B})$ and $P(A \cup B)$.
c. Are the events A and B mutually exclusive? Explain.

a. $P(A) = P(E_1) + P(E_4) + P(E_6)$

2 $= 0.05 + 0.25 + 0.1$

$= 0.40$

1 $P(A \cap B) = P(E_4) = 0.25$

b. $P(\bar{B}) = P(\{E_1, E_3, E_5, E_6\})$

2 $= 0.05 + 0.20 + 0.15 + 0.10$

$= 0.50$

OR $P(\bar{B}) = 1 - P(B)$

$= 1 - [0.2 + 0.25 + \ldots$

$= 1 - 0.50$

$= 0.50$

2 $P(A \cup B) = P[\{E_1, E_2, E_4, E_6, E_7\}]$

$= 1 - [P(E_3) + P(E_5)]$

$= 1 - (0.2 + 0.15) = 0.65$

c. A and B are not mutually exclusive, since

2 $A \cap B = E_4 \neq \phi$.

Q7. (6 Marks)
Weekly demand at a grocery store for a brand of breakfast cereal is normally distributed with a mean of 800 boxes and a standard deviation of 75 boxes.

a. What is the probability that weekly demand is between 725 and 950 boxes?
b. The store orders cereal from a distributor weekly. How many boxes should the store order for a week to have only a 5% chance of running short of this brand of cereal during the week?

X = weekly demand ; $X \sim N(800, (75)^2)$

$$P(725 \le X \le 950) = P\left(\frac{725-800}{75} \le \frac{X-800}{75} \le \frac{950-800}{75}\right) \quad 2$$

$$= P(-1 \le Z \le 2) \quad \text{where } Z \sim N(0,1)$$

$$= 0.4772 + 0.3413 \quad 1$$

$$= 0.8185$$

Find x_0 such that $P(X \ge x_0) = 0.05$

$$P\left(\frac{X-800}{75} \ge \frac{x_0-800}{75}\right) = 0.05 \quad 2$$

Since $P(Z \ge 1.645) = 0.05$

$$\Rightarrow \frac{x_0-800}{75} = 1.645 \Rightarrow x_0 = (1.645)(75) + 800 = 923.375$$

Store should order 924 Boxes 1

Q8. (5 Marks)
There are 8 flights daily from Riyadh to Dammam. The probability that any flight arrives late is 0.20, and that flight arrivals are independent.

a. What is the probability that at least 2 flights arrive late?
b. How many flights do we expect to arrive late daily?

X = # of flights arriving late ; $X \sim B(8, 0.20)$

a. 1 $$P(X \ge 2) = 1 - P(X < 2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

1 $$= 1 - \left[\binom{8}{0}(0.2)^0(0.8)^8 + \binom{8}{1}(0.2)(0.8)^7\right]$$

1 $$= 1 - (0.167 + 0.335) = 0.498$$

2 b. Expected Number of late flights $= (0.2)(8) = 1.6$ flig