Example 4.1 Life length of a particular type of battery follows exponential distribution with mean 2 hundred hours. Find the probability that the

(a) life length of a particular battery of this type is less than 2 hundred hours.
(b) life length of a particular battery of this type is more than 4 hundred hours.
(c) life length of a particular battery of this type is less than 2 hundred hours or more than 4 hundred hours.

Solution Let X = life length of a battery. Then $X \sim Exp(1/2)$, by $\mu = 1/\lambda = 2$ (given) so that $\lambda = 1/2 = 0.5$

(a) $P(X < 2) = F(2) = 1 - e^{-(0.5)2} = 1 - e^{-1} = 0.632$

(b) $P(X > 4) = 1 - P(X \le 4) = 1 - \left[1 - e^{0.5(4)}\right] = e^{-2} = 0.135$

(c) $P\left[(X < 2) \text{ or } (X > 4)\right] = P(X < 2) + P(X > 4) = 1 - e^{-1} + e^{-2} = 0.767$

Q: Given that the switchboard of a consultant's office receives on the average 0.6 calls per minute. Find the probabilities that the time between the successive calls arriving at the switchboard of the consulting firm will be

(a) less than ½ minute;
(b) **more than 3 minutes.**
(c) **Between ½ and 5 min**

Exercises

4.1 The tread wear (in thousands of kilometers) that car owners get with a certain kind of tire is a random variable whose probability density is given by

$$f(x) = \frac{1}{30}\ e^{-x/30} \qquad 0 \le x < \infty$$

(a) Find the probability that one of these tires will last at most 18000 kilometers.
(b) Find the probability that one of these tires will last anywhere from 27000 to 36000 kilometers.

4.2 A transistor has an exponential time to failure distribution with mean time to failure of β = 20,000 hours.

(a) What is the probability that the transistor fails by 30,000 hours?
(b) The transistor has already lasted 20, 000 hours in a particular application. What is the probability that it fails by 30, 000 hours?

4.3 The lifetime X (in hours) of the central processing unit of a certain type of microcomputer is an exponential random variable with parameter 0.001. What is the probability that the unit will work at least 1,500 hours?

4.4 The lifetime (in hours) of the central processing unit of a certain type of microcomputer is an exponential random variable with mean 1000.

(a) What is the probability that a central processing unit will have a lifetime of at least 2000 hours?
(b) What is the probability that a central processing unit will have a lifetime of at most 2000 hours

4.5 The amount of raw sugar that one plant in a sugar refinery can process in one day can be modeled as having an exponential distribution with a mean of 4 tons. What is the probability that any plant processes more than $4\,ln2$ tons of sugar on a day?

4.6 The amount of time that a surveillance camera will run without having to be rested is a random variable having the exponential distribution with $\lambda = 50$ days. Find the probabilities that such a camera will

(a) have to rested in less than 20 days;
(b) not have to rested in at least 60 days

4.7 Consider a random variable having the exponential distribution with parameter λ = 0.25. Find the probabilities that

(a) it takes values more than 200;
(b) it takes values less than 300.

4.8 If on the average three trucks arrive per hour to be unloaded at a warehouse. Find the probability that the time between the arrivals of successive trucks will be less than 5 minutes.

4.9 The number of weekly breakdowns of a computer is a random variable having a Poisson distribution with $\lambda = 0.3$. Find the percent of the time that the interval between the breakdowns of the computer will be

(a) less than one week;

(b) at least 5 weeks.