

1. (8 Marks) Lube-Tech is a major chain whose primary business is performing lube and oil changes for passenger cars. The national operations manager has stated in an industry newsletter that the mean number of miles between oil changes for all passenger cars exceeds 4,200 miles. To test this, an industry group has selected a random sample of 100 cars that have come into a lube shop and determined the number of miles since the last oil change and lube. The sample mean was 4,278 and the sample standard deviation was 780 miles. Based on a significance level of 0.10 is the company claim acceptable? Clearly state your hypotheses, p-value and your conclusions.

$$\textcircled{1} \begin{array}{lll} H_0: \mu \leq 4200 & n = 100 & s = 780 \\ H_A: \mu > 4200 & \bar{x} = 4278 & \alpha = 0.1 \end{array}$$

Assumptions:

$$\textcircled{1} \left. \begin{array}{l} \textcircled{1} \text{ Large Sample} \\ \textcircled{2} \text{ Unknown } \sigma \end{array} \right\} \Rightarrow \text{use } z_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$z_0 = \frac{4278 - 4200}{780/\sqrt{100}} = \frac{(78)10}{780} = 1 \quad \textcircled{1}$$

Since it is right-tailed

$$p\text{-value} = P(Z > z_0) = P(Z > 1) = 0.1587 \quad \textcircled{2}$$

Since $p\text{-value} > \alpha = 0.1 \Rightarrow$ Do NOT reject H_0 . $\textcircled{2}$

Conclusion

The company claim is NOT acceptable. $\textcircled{1}$

2. (10 Marks) A Company makes a soft drink dispensing machine that allows customers to get soft drinks from the machine in a cup with ice. When the machine is running properly, the average number of fluid ounces in the cup should be 14. Periodically the machines need to be tested to make sure that they have not gone out of adjustment. To do this, six cups are filled by the machine and a technician carefully measures the volume in each cup. In one such test, the following data were observed:

14.25	13.7	14.02
14.13	13.99	14.04

Using 5% level of significance, do you think that the company claim is justified?

① $H_0: \mu = 14$ $n = 6$
 $H_a: \mu \neq 14$ $\bar{x} = 14.022$ $s = 0.184$ $\alpha = 0.05$

Assumptions:

- ① Small sample.
 ② Assume normal pop.
 ③ Unknown σ
- \Rightarrow use $t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

$$t_0 = \frac{14.022 - 14}{0.184/\sqrt{6}} = \frac{(0.022)\sqrt{6}}{0.184} = 0.293 \quad ①$$

$$t_{\frac{\alpha}{2}; n-1} = t_{0.05, 5} = 2.5706 \quad ①$$

Since $|t_0| < |t_{\frac{\alpha}{2}}| \Rightarrow$ Do NOT reject H_0 . ②

Conclusion:

The company claim is justified. ②

3. (8 Marks) A Bank is interested to see whether there is a difference between average daily balances in checking accounts that are joint accounts (two or more members per account) versus single accounts (one member per account). To test this, random samples of checking accounts was selected with the following results:

Single Accounts

$$n_1 = 20$$

$$s_1 = \$256$$

$$\bar{x}_1 = \$1,123$$

Joint Accounts

$$n_2 = 30$$

$$s_2 = \$300$$

$$\bar{x}_2 = \$1,245$$

Based upon these data, using 5% level of significance, test whether the two populations have equal means. State the assumptions you used to conduct this test.

$$\textcircled{1} \quad H_0: \mu_1 = \mu_2 \quad \text{or} \quad H_0: \mu_1 - \mu_2 = 0 \quad \alpha = 0.05$$

$$H_A: \mu_1 \neq \mu_2 \quad \text{or} \quad H_A: \mu_1 - \mu_2 \neq 0$$

Assumptions:

- ① Samples are independent
- ② Small samples.
- ③ Assume normal pops.
- ④ Unknown σ 's.
- ⑤ Assume Equal σ 's.

$$t_0 = \frac{(\bar{x}_2 - \bar{x}_1) - 0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \quad s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= 283.4 \quad \textcircled{1}$$

$$t_0 = \frac{1245 - 1123}{(283.4) \sqrt{\frac{1}{20} + \frac{1}{30}}} = \frac{122}{81.84} = 1.491 \quad \textcircled{1}$$

$$t_{\frac{\alpha}{2}, n_1 + n_2 - 2} = t_{0.025, 48} \approx t_{0.025, 50} = 2.009 \quad \textcircled{1}$$

Since $|t_0| < |t_{\frac{\alpha}{2}}| \Rightarrow$ Do NOT Reject H_0 $\textcircled{2}$

Conclusion

The two means are ~~not~~ equal. $\textcircled{1}$

4. (8 Marks) There are a number of highly touted search engines for finding things of interest on the Internet. Recently, a consumer rating system ranked two search engines ahead of the others. Now, a computer user's magazine wishes to make the final determination regarding which one is actually better at finding particular information. To do this, each search engine was used in an attempt to locate specific information using specified key words. Both search engines were subjected to 100 queries. Search engine 1 successfully located the information 88 times and search engine 2 located the information 80 times. Using a significance level equal to 0.05, the magazine editor think that search engine 1 is better than search engine 2, do you agree with him? Clearly state your hypotheses and use the p-value method to test this hypothesis stating the necessary assumptions for this test.

$$\begin{aligned} \textcircled{1} \quad H_0: p_1 \leq p_2 \quad \text{or} \quad H_0: p_1 - p_2 \leq 0 \quad \alpha = 0.05, \\ H_A: p_1 > p_2 \quad \text{or} \quad H_A: p_1 - p_2 > 0 \quad n_1 = n_2 = 100 \\ \bar{p}_1 = 0.88, \quad \bar{p}_2 = 0.8 \end{aligned}$$

Assumption s:

- ① $n_1 = n_2 = 100 \geq 30$
- ② $n_1 p_1 = 88 \geq 5$ and $n_1 (1 - p_1) = 12 \geq 5$
- ③ $n_2 p_2 = 80 \geq 5$ and $n_2 (1 - p_2) = 20 \geq 5$

$$z_0 = \frac{(\bar{p}_1 - \bar{p}_2) - 0}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad \bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{88 + 80}{200} = 0.84 \quad \textcircled{1}$$

$$z_0 = \frac{(0.88 - 0.8) - 0}{\sqrt{(0.84)(0.16)\left(\frac{2}{100}\right)}} = \frac{(0.08)(10)}{\sqrt{2(0.84)(0.16)}} = 1.54 \quad \textcircled{1}$$

$$z_{\alpha} = z_{0.05} = 1.645$$

$$p\text{-value} = P(Z > z_0) = P(Z > 1.54) = 0.0618 \quad \textcircled{2}$$

Since p-value $\nless \alpha \Rightarrow$ Do NOT reject H_0 . $\textcircled{1}$

Conclusion

We do NOT agree with the magazine editor. $\textcircled{1}$

5. (8 Marks) One of the major automobile makers has developed two new engines. At question is whether the two engines have the same variability with respect to miles per gallon. To test this, the following information is available:

Engine 1	Engine 2
$n_1 = 7$	$n_2 = 9$
$\bar{x}_1 = 28.7$	$\bar{x}_2 = 33.4$
$s_1 = 3.4$	$s_2 = 4.12$

Based on this situation and the information provided, do you believe that there is significance in the variability of the two engines? Explain, using 5% level of significance.

$$\textcircled{1} \quad H_0: \sigma_1^2 = \sigma_2^2 \quad \text{or} \quad H_0: \sigma_1^2 - \sigma_2^2 = 0 \quad \alpha = 0.05$$

$$H_A: \sigma_1^2 \neq \sigma_2^2 \quad \text{or} \quad H_A: \sigma_1^2 - \sigma_2^2 \neq 0$$

Assumptions:

① Two populations are indep.

② Two populations are normal.

$$\textcircled{1} \quad F_0 = \frac{s_2^2}{s_1^2}, \quad df_1 = 8, \quad df_2 = 6$$

$$F_0 = \frac{(4.12)^2}{(3.4)^2} = 1.47$$

$$F_{\frac{\alpha}{2}, n_2-1, n_1-1} = F_{0.025, 8, 6} = 5.6 \quad (\text{use } 4.147)$$

Since $F_0 < F_{\frac{\alpha}{2}} \Rightarrow$ Do NOT reject H_0 .

① Conclusion:

We do NOT believe that there is a significant difference in the variability of the two engines.

6. (8 Marks) The Crack-it Nut and Candy Company wishes to control their packaging so that the standard deviation in fill volume for their 6-ounce packages does not exceed .09 ounces. To monitor this, they regularly select random samples of $n = 20$ packages and found that the sample standard deviation is 0.13 ounces. If they desire a level of significance of 5%, test the hypothesis and what is your final conclusion?

$$\begin{aligned} H_0: \sigma^2 &\leq 0.0081 & n &= 20 & \alpha &= 0.05 \\ \textcircled{1} H_A: \sigma^2 &> 0.0081 & s &= 0.13 & s^2 &= 0.0169 \end{aligned}$$

Assumptions:

- ① Population is normal.

$$\textcircled{1} \chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{19(0.0169)}{0.0081} = 39.642 \textcircled{1}$$

$$\chi^2_{\alpha, n-1} = \chi^2_{0.05, 19} = 30.14 \textcircled{1}$$

since $\chi^2 > \chi^2_{\alpha} \Rightarrow$ Reject H_0 ①

Conclusion:

~~The~~ The standard deviation in fill volume ①

EXCEEDS 0.09 ounces -