KING FAHD UNIVERSITY OF PETROLUEM AND MINERALS

Math 001 - Term 041

First Test

Name: ID#: Sec#: 3 & 9 & 15 Sr#:

Q1. **(5 Points)** $A=\left\{-\sqrt{16},-\frac{p}{2},\frac{-3}{2},\sqrt{5},0,1,51,0.33\cdots,2.31253\cdots,9,7\right\}$ then A has

a) ...8.....rational numbers.
b) ...3.....irrational numbers.
c) ...1.....prime numbers.
d) ...2....composite numbers.
e) ...6...integer numbers.

Q2. **(5 Points)** If $A=\left(\frac{-27}{8}\right)^{-\frac{1}{3}}$, $B=\sqrt[3]{0.027}$, then find AB.

$$A=\left(\frac{-27}{8}\right)^{-\frac{1}{3}}=\frac{(-8)^{\frac{1}{3}}}{(27)^{\frac{1}{3}}}=\frac{\left(-2^3\right)^{\frac{1}{3}}}{\left(3^3\right)^{\frac{1}{3}}}=\frac{-2}{3}$$

$$B=\sqrt[3]{0.027}=\sqrt[3]{(0.3)^3}=0.3$$

$$AB=\frac{-2}{3}\cdot\frac{3}{10}=\frac{-2}{10}=\frac{-1}{5}$$

Q3. **(2 Point)** Express "the real number x is more then 2 units from 4 but less than 7 units from4" by using absolute value notation.

$$2<|x-4|<7$$

Q4. **(5 Points)** Simplify $5x\sqrt[3]{54x^4}-3\sqrt[3]{16x^7}$. Write the answer in the simplest form.

$$5x\sqrt[3]{54x^4}-3\sqrt[3]{16x^7}=5x\sqrt[3]{2\cdot3^3x^3x}-3\sqrt[3]{2\cdot2^3\left(x^2\right)^3x}$$
$$=15x^2\sqrt[3]{2x}-6x^2\sqrt[3]{2x}=9x^2\sqrt[3]{2x}$$

Q5. **(5 Points)** Find the coefficient of a^{3x} in the product $\left(a^x-1\right)\left(2a^x+2\right)^3$

$$\left(a^x-1\right)\left(2a^x+2\right)^3=\left(a^x-1\right)\left(8a^{3x}+24a^{2x}+24a^x+8\right)$$
$$=8a^{4x}+24a^{3x}+24a^{2x}+8a^x-8a^{3x}-24a^{2x}-24a^x-8$$
$$=8a^{4x}+16a^{3x}-16a^x-8$$

coefficient of $a^{3x}=16$

Q6. **(10 Points)** Simplify

a) $\left(\frac{x^3-1}{x^2+x+1}-\frac{x^2-1}{x-1}\right)\div\frac{x-1}{-x^2+5x-6}$

$$\text{The expression}=\left(\frac{(x-1)\left(x^2+x+1\right)}{x^2+x+1}-\frac{(x-1)(x+1)}{x-1}\right)\div\frac{x-1}{-(x-3)(x-2)}$$

$$=\left(\frac{(x-1)}{1}-\frac{(x+1)}{1}\right)\div\frac{x-1}{-(x-3)(x-2)}=-(x-1-x-1)\cdot\frac{(x-3)(x-2)}{x-1}$$

$$=\frac{2(x-3)(x-2)}{x-1}$$

b) $\frac{x}{(x-1)^2}-\frac{2x}{x^2-1}+\frac{1}{x+1}$

$$\textit{The Expression}=\frac{x}{(x-1)^2}-\frac{2x}{(x-1)(x+1)}+\frac{1}{x+1}=\frac{x(x+1)-2x(x-1)+(x-1)^2}{(x-1)^2(x+1)}$$

$$=\frac{x(x+1)-2x(x-1)+(x-1)^2}{(x-1)^2(x+1)}=\frac{x^2+x-2x^2+2x+x^2-2x+1}{(x-1)^2(x+1)}$$

$$=\frac{x+1}{(x-1)^2(x+1)}=\frac{1}{(x-1)^2}$$

Q7. **(9 Points)** Factor the following completely

a) $$x^{4n}-2x^{2n}+1=\left(x^{2n}\right)^2-2x^{2n}+1=\left(x^{2n}-1\right)\left(x^{2n}-1\right)$$
$$=\left(\left(x^n-1\right)\left(x^n+1\right)\right)^2$$

b) $$a^2-c^2+10ab+25b^2=\left(a^2+10ab+25b^2\right)-c^2=(a+5b)^2-c^2$$
$$=(a+5b+c)(a+5b-c)$$

c) $$x^2y^2-1-2xyz+z^2=\left(x^2y^2-2xyz+z^2\right)-1$$
$$=(xy-z)^2-1$$
$$=(xy-z+1)(xy-z-1)$$

Q8. **(6 Points)** Simplify the following by rationalizing the denominator.

a) $\dfrac{2}{\sqrt[5]{9x^3y}}$

$$\textit{The Expression} = \frac{2}{\sqrt[5]{3^2x^3y}} \cdot \frac{\sqrt[5]{3^3x^2y^4}}{\sqrt[5]{3^3x^2y^4}} = \frac{2\sqrt[5]{3^3x^2y^4}}{\sqrt[5]{3^5x^5y^5}} = \frac{2\sqrt[5]{3^3x^2y^4}}{3xy}$$

b) $\dfrac{2h}{\sqrt{x+h}-\sqrt{x}}, \; h \neq 0$

$$\textit{The Expression} = \frac{2h}{\sqrt{x+h}-\sqrt{x}} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{2h\left(\sqrt{x+h}+\sqrt{x}\right)}{x+h-x} = 2\left(\sqrt{x+h}+\sqrt{x}\right)$$

Q9. **(5 Points)** Find the product of $\left(\sqrt[3]{x+2}-\sqrt[3]{x-2}\right)\cdot\left(\sqrt[3]{(x+2)^2}+\sqrt[3]{x^2-4}+\sqrt[3]{(x-2)^2}\right)$

$$\textit{TheExpression} = \left(\sqrt[3]{x+2}\right)^3 - \left(\sqrt[3]{x-2}\right)^3 = x+2-x+2 = 4$$

Q10. **(4 Points)**

By rationalizing the ***denominators***, simplify the expression $\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+2}+\dfrac{1}{2+\sqrt{5}}$

$$\textit{The Expression} = \left(\frac{1}{\sqrt{2}+\sqrt{3}}\right)\cdot\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+2}\right)\cdot\left(\frac{\sqrt{3}-2}{\sqrt{3}-2}\right)+\left(\frac{1}{2+\sqrt{5}}\right)\left(\frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$

$$= \left(\frac{\sqrt{2}-\sqrt{3}}{2-3}\right)+\left(\frac{\sqrt{3}-2}{3-4}\right)+\left(\frac{2-\sqrt{5}}{4-5}\right) = -\sqrt{2}+\sqrt{3}-\sqrt{3}+2-2+\sqrt{5} = \sqrt{5}-\sqrt{2}$$

Q11. **(5 Points)** Simplify $\left(\frac{\sqrt{x}\sqrt[3]{y^2}}{x}\right)^{12}\left(\frac{x^{\frac{1}{2}}y}{x^{\frac{-1}{2}}\sqrt[4]{z}}\right)^{-2}$

$$\textit{The Expression} = \left(\frac{\sqrt{x}\sqrt[3]{y^2}}{x}\right)^{12}\left(\frac{x^{\frac{1}{2}}y}{x^{\frac{-1}{2}}\sqrt[4]{z}}\right)^{-2} = \left(x^{\frac{1}{2}-1}y^{\frac{2}{3}}\right)^{12}\left(\frac{x^{\frac{1}{2}+\frac{1}{2}}y}{z^{\frac{1}{4}}}\right)^{-2}$$

$$= \left(x^{\frac{-1}{2}}\right)^{12}\left(y^{\frac{2}{3}}\right)^{12}\frac{x^{-2}y^{-2}}{\left(z^{\frac{1}{4}}\right)^{-2}} = \left(x^{-6}y^{8}\right)\left(x^{-2}y^{-2}\right)z^{\frac{1}{2}} = x^{-8}y^{6}\sqrt{z} = \frac{y^{6}\sqrt{z}}{x^{8}}$$

Q12. **(3 Points)** Solve $\frac{x}{x-4}+5=\frac{10}{x-4}$

$$\frac{x}{x-4}-\frac{10}{x-4}=-5$$

$$\frac{x-10}{x-4}=-5$$

$$x-10=-5x+20$$

$$x+5x=20+10$$

$$6x=30$$

$$x=5$$

With my best wishes