KING FAHD UNIVERSITY OF PETROLUEM AND MINERALS

Math 001 - Term 041

First Test

Name: ID#: Sec#: Sr#:

Q1. **(2 Points)** Solve the formula $\frac{X}{Y}=\frac{A-B}{B-C}$ for B

$$\frac{X}{Y}=\frac{A-B}{B-C}\Rightarrow XB-XC=AY-BY\Rightarrow BX+XY=AY+XC$$

$$B(X+Y)=AY+XC\Rightarrow B=\frac{AY+XC}{X+Y}$$

Q2. **(3 Points)** if $y>0$, Simplify $\left[\frac{(-2y)^0 y^{-2}(5y^2)^{-3}}{y^{-3}(5^{-1}y^5)^{-1}}\right]^{\frac{-1}{4}}$

$$\left[\frac{(-2y)^0 y^{-2}(5y^2)^{-3}}{y^{-3}(5^{-1}y^5)^{-1}}\right]^{\frac{-1}{4}}=\left[5^{-3-1}.y^{-2-6+3+5}\right]^{\frac{-1}{4}}=\left[5^{-4}y^0\right]^{\frac{-1}{4}}=5$$

Q3. **(3 Points)** If $1<x<2$, then write $|5x-3|-|x-2|$ **without absolute value notation**.

$$\underbrace{|5x-3|}_{+ve}-\underbrace{|x-2|}_{-ve}=(5x-1)-(2-x)=5x-3-2+x=6x-5$$

Q4. **(3 Points)** if $x<y$ Simplify $\sqrt{x^2-2xy+y^2}+\sqrt[3]{(x-y)^3}$

$$\sqrt{x^2-2xy+y^2}+\sqrt[3]{(x-y)^3}=\sqrt{(x-y)^2}+\sqrt[3]{(x-y)^3}$$
$$=|x-y|+(x-y)=y-x+x-y=0$$

Q5. **(3 Points)** Write $\frac{(0.00006)\cdot(7000000)}{21\times10^{-7}}$ in **scientific** notation

$$\frac{(0.00006)\cdot(7000000)}{21\times10^{-7}}=\frac{(6\times10^{-5})(7\times10^{6})}{21\times10^{-7}}=\frac{42\times10}{21\times10^{-7}}=2\times10^{8}$$

Q6. **(6 Points)** Simplify

a) $\dfrac{1}{2x-1}-\dfrac{x+1}{6x^2-5x+1}\div\dfrac{x^2+3x+2}{3x^2+2x-1}$

$$=\frac{1}{2x-1}-\frac{x+1}{(2x-1)(3x+1)}\cdot\frac{(3x-1)(x+1)}{(x+2)(x+1)}$$

$$=\frac{1}{2x-1}-\frac{x+1}{(2x-1)(x+2)}=\frac{x+2-x-1}{(2x-1)(x+2)}=\frac{1}{(2x-1)(x+2)}$$

b) $\dfrac{3+\sqrt{3}}{3-\sqrt{3}}-\dfrac{3-\sqrt{3}}{3+\sqrt{3}}$

$$\frac{3+\sqrt{3}}{3-\sqrt{3}}-\frac{3-\sqrt{3}}{3+\sqrt{3}}=\frac{(3+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}-\frac{(3-\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$

$$=\frac{(3+\sqrt{3})^2}{9-3}-\frac{(3-\sqrt{3})^2}{9-3}=\frac{9+6\sqrt{3}+3-9+6\sqrt{3}-3}{6}=\frac{12\sqrt{3}}{6}=2\sqrt{3}$$

Q7. **(9 Points)** Factor the following **completely**

a) $4+4x+x^2-y^2$

$$4+4x+x^2-y^2=(4+4x+x^2)-y^2=(2+x)^2-y^2=(2+x-y)(2+x+y)$$

b) $27x^4-108x^3-8x+32$

$$27x^4-108x^3-8x+32=27x^3(x-4)-8(x-4)$$
$$=(x-4)(27x^3-8)=(x-4)(3x-2)(9x^2+6x+4)$$

c) $3x^2+xy-2y^2-x-y$

$$3x^2+xy-2y^2-x-y=(3x^2+xy-2y^2)-x-y$$
$$(3x-2y)(x+y)-(x+y)=(x+y)(3x-2y-1)$$

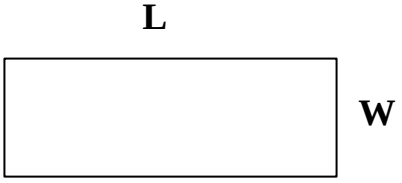
Q8. **(3 Points)** If $z=\dfrac{(2i-1)^2-\sqrt{-2}\sqrt{-8}}{2+i^{103}}$, where $i=\sqrt{-1}$, then Find $5\overline{z}$

$$z=\frac{(2i-1)^2-\sqrt{-2}\sqrt{-8}}{2+i^{103}}=\frac{4i^2-4i+1+4}{2-i}=\frac{-4-4i+1+4}{2-i}=\frac{1-4i}{2-i}$$
$$=\frac{(1-4i)}{(2-i)}\cdot\frac{(1+i)}{(2+i)}=\frac{2+i-8i+4}{5}=\frac{6-7i}{5}=\frac{6}{5}-\frac{7}{5}i$$

The Conjugate of z is $\bar{z}=\dfrac{6}{5}+\dfrac{7}{5}i\Rightarrow 5\bar{z}=6+7i$

Q9. **(5 Points)**

The length of a rectangle is **4 cm** more than the width of the rectangle. If the area of this rectangle is **60 cm²**, then find the length and the width.



Let L :Length and W :width

$L=W+4$

Area $=LW$

$60=LW$

$$(W+4)W=60\Rightarrow W^2+4W-60=0\Rightarrow(W+10)(W-6)=0$$

Then $W=-10$ Rejected

$$W=6\ cm\Rightarrow L=W+4=6+4=10$$

Q10. **(5 Points)**

If the **discrimenant** of the equation $x^2+4x+(k-1)=0$ is equal to **8**, then find the roots of the equation

$$Discrimenant=b^2-4ac=16-4(1)(k-1)=8$$

$$16-4k+4=8\Rightarrow -4k=-12\Rightarrow k=3$$

$$\therefore x^2+4x+2=0$$

$$x=\frac{-4\pm\sqrt{8}}{2}\ \frac{-4\pm2\sqrt{2}}{2}=-2\pm2\sqrt{2}$$

Q11. **(3 Points)**

After **completing the square** the equation $4x^2-3x+15=x$, getting $(x-a)^2=b$, Find $\frac{a}{b}$:

$4x^2-3x+15=x$

$4x^2-4x+15=0 \Rightarrow x^2-x+\frac{15}{4}=0 \Rightarrow x^2-x+\left(\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2-\frac{15}{4}=\left(x-\frac{1}{2}\right)^2=\frac{-7}{2}$

$a=\frac{1}{2}$ *and* $b=\frac{-7}{2} \Rightarrow \frac{a}{b}=\frac{1/2}{-7/2}=\frac{1}{2}\cdot\frac{-2}{7}=\frac{-1}{7}$

Q12. **(3 Points)**

Find the solution set of the equation $|3-|1-3x||=5$

$|3-|1-3x||=5$

$3-|1-3x|=5 \quad or \quad 3-|1-3x|=-5$

$-|1-3x|=2 \quad or \quad -|1-3x|=-8$

$|1-3x|=-2$ (*rejected*)

$or \ |1-3x|=8 \Rightarrow 1-3x=8 \quad or \quad 1-3x=-8$

$-3x=7 \quad or \quad -3x=-9$

$x=\frac{-7}{3} \quad or \quad x=3$

Q13. **(3 Points)** Simplify the expression $\sqrt{\sqrt[3]{64x^{10}y^6}}$

$\sqrt{\sqrt[3]{64x^{10}y^6}}=\sqrt[6]{2^6x^6y^6}\cdot\sqrt[6]{x^4}=2|xy|x^{\frac{4}{6}}=2|xy|x^{\frac{2}{3}}=2|xy|\sqrt[3]{x^2}$

Q14. **(3 Points)** Solve $\frac{3}{x-2}-\frac{4}{2x+1}=\frac{1}{x-2}$

Multiply by $(x-2)(2x+1)$, *we get*

$3(2x+1)-4(x-2)=(2x+1)$

$6x+3-4x+8=2x+1$

$2x+11=2x+1$

$11=1$

$10=0$

No solution

The set solution is Φ