

## Questions from old Exams

### 1 Section 5.1

1. If a hard disk in a computer rotates at 2700 revolutions per minute, then find the angular speed of the disk in radians per second.
2. Find the length of an arc that subtends a central angle  $240^\circ$  in a circle of radius 15 cm.
3. Find the length of an arc that subtends a central angle  $150^\circ$  in a circle of diameter 14 cm.
4. Find the measure of the complement of the angle  $54^\circ 23' 37''$ .
5. Find the length of an arc that subtends a central angle  $80^\circ$  in a circle of radius 10 cm.
6. A car with a wheel of radius 13 inches is moving with a speed of 50 mph. Find the angular speed of the wheel in radian per minute.
7. A wheel of a truck has a radius 1.6 feet.
  - (a) How far will the truck move if the wheel turns through  $40^\circ$ ?
  - (b) If the wheel is rotating at the rate of 6 revolutions per second, find the speed of the truck in feet per second.
8. If  $\alpha = \frac{\pi}{3}$  radians and  $\beta$  is the complementary angle of  $\alpha$ , then find  $150^\circ - \beta$ .
9. If a central angle in a circle of diameter 10 cm has a measure of 1 radian, then find the length of the arc that intercepts with the angle.
10. Find the angle  $\theta$  such that the angle  $\theta$  and the angle  $150^\circ 50' 23''$  are supplementary.
11. Convert the angle  $\frac{\pi}{18}$  radian to revolutions.
12. A wheel on a truck has a diameter of  $\frac{12}{\pi}$  feet. How far does the truck move as the wheel turns through an angle  $105^\circ$ ?
13. The wheels on a truck turn at the rate of  $k$  revolutions per second when the truck is travelling at 60 feet per second. If the diameter of the wheel is 3 feet, what is the value of  $k$ ?
14. Find the smallest positive angle coterminal with the angle  $\theta = -725^\circ$ .

15. Find the radian measure of the angle  $\theta$  that is complementary to the angle  $58^\circ 45'$ .
16. A bicycle has wheels with radius 10 inches. If the wheel make 90 revolutions per minute, then find the linear speed of he bicycle in inches per minute.
17. Find the radian measure of the central angle which cuts off an arc of length 6 inches on a circle of radius 5 inches.
18. Find the length of the arc on a circle of radius 6 cm cut by a central angle of  $120^\circ$ .
19. Find the measure of the angle  $31^\circ 15'$  in radians.
20. Find the length of the arc on a circle of diameter 12 cm cut by a central angle of  $40^\circ$ .
21. Convert  $36^\circ$  into radians and  $\frac{7\pi}{45}$  radians into degrees.
22. Find the degree measure of a central angle which cuts off an arc of length 10 cm in a circle of radius 6 cm.
23. Find the length of the smaller arc of a circle of radius 1 cm whose angle is  $90^\circ$ .
24. Find the smallest positive angle coterminal with  $-743^\circ$ .
25. Find the degree measure of a central angle that cuts off an arc length  $7\pi$  cm on a circle of radius 3 cm.
26. Suppose that a point  $P$  is on the circle of radius 2 cm and center  $O$ , and the ray  $OP$  is rotating with angular velocity  $\frac{\pi}{6}$  radians per second. Find the linear velocity of  $P$ .
27. Which one of the following is not coterminal with  $50^\circ$ ?
  - (a)  $410^\circ$
  - (b)  $230^\circ$
  - (c)  $-310^\circ$
  - (d)  $770^\circ$
  - (e)  $-670^\circ$
28. A wheel is rotating at 100 revolutions per minute. Find the angular speed in radian per second.

## 2 Section 5.2

1. A 22 feet ladder is resting against a wall and makes an angle of  $60^\circ$  with the ground. Find the exact height to which the ladder will reach the wall.
2. Find the exact value of a)  $\sin^2 45^\circ + \cos^2 60^\circ$       b)  $3 \cot \frac{\pi}{6} + \sec \frac{\pi}{4} \sin 60^\circ$ .
3. A 8 foot ladder is resting against a wall and makes an angle of  $\frac{\pi}{6}$  with the ground. Find the exact height to which the ladder will reach the wall.
4. The top of a radio antenna is 100 m high from the ground. A wire 200 m long is attached to the top from the ground. Find the angle the wire makes with the ground.
5. In the adjacent figure, the angle of elevation measured from a point on the ground to the top of a pole which is 10 meters high is found to be  $30^\circ$ . The measurement is taken again after moving  $x$  meters closer to the pole and is found to be  $45^\circ$ . What is the value of  $x$ ?
6. A man stands 10 feet from a tree and measures the angle of elevation to the top of the tree as  $60^\circ$ . He moves back from the tree until the angle becomes  $30^\circ$ . How far did he move back?
7. The following figure shows that two ships are at points  $P$  and  $Q$  and also, an airplane is at point  $R$ . When the airplane is at the height of 3500 feet, the angle of depression from airplane to  $P$  is  $45^\circ$  and to  $Q$  is  $30^\circ$ . Find the distance between the two ships.

## 3 Section 5.3

1. If  $\cos 2 = m$  and  $\sin 2 = n$ , then  $n - m$  is a) a positive real number.      b) a negative real number      c) zero      d) undefined.
2. Find the exact value of
  - (a)  $\cos\left(\frac{11\pi}{6}\right) - 4 \sin\left(\frac{2\pi}{3}\right)$ .
  - (b)  $\sec \frac{7\pi}{6} + 6 \cot \frac{4\pi}{3}$ .
3. On the terminal side of a central angle  $\theta$ , a point  $P$  has the coordinates  $(3, -4)$ . What is the value of  $\tan \theta - \sin \theta$ ?
4. If the terminal side of the angle  $\theta$  lies on the line  $3x + 4y = 0$  and  $x < 0$ , then find the value of  $\sin \theta + \cos \theta + \tan \theta$ .

5. Which of the following statements is TRUE?
- (a) If  $45^\circ < x < 90^\circ$ , then  $\cos 3x$  is negative.  
 (b)  $\tan 2 > \tan 1$ , ( angles in radians).  
 (c) The range of  $y = \sin x$  is the open interval  $(-1, 1)$ .  
 (d) If  $45^\circ < x < 90^\circ$ , then  $\cot 2x$  is positive.
6.  $\tan 920^\circ =$       a)  $-\tan 60^\circ$       b)  $-\tan 200^\circ$       c)  $\tan 40^\circ$       d)  $\tan 20^\circ$       e)  $\tan 30^\circ$ .
7. Find the value of
- (a)  $\frac{\cos 60^\circ + \sin 270^\circ + \sec 240^\circ}{(\tan 135^\circ)(\sec 150^\circ) - (\csc 90^\circ)(\cot 30^\circ)}$ .  
 (b)  $\frac{\tan 225^\circ + \csc 150^\circ - \tan 60^\circ}{\sec 300^\circ + \cot 210^\circ}$ .  
 (c)  $\cot \frac{41\pi}{3}$ .  
 (d)  $\cot 225^\circ + \sec 150^\circ - \tan 60^\circ$ .
8. Which one of the following is undefined?  
 a)  $\cos(\ln 1)$       b)  $\ln(\cos \pi)$       c)  $\sin(\ln e)$       d)  $\log(\ln e^2)$       e)  $\log(\cos 0)$
9. If  $(-1, -\frac{4}{3})$  is on the terminal side of angle  $\theta$  in standard position, then find  $\sec \theta - \csc \theta$ .
10. Suppose that  $90^\circ < \theta < 180^\circ$ , then find the signs of  $\sin 2\theta$ ,  $\tan \frac{\theta}{2}$ , and  $\cos \theta$ .
11. Given  $\cos x = -\frac{3}{5}$ . Find the largest possible value of  $\frac{\sec x - \tan x}{\sin x}$ .
12. Which one of the following is TRUE?      a)  $\tan 3 > \sin 3$       b)  $\sin 3 = \tan 3$       c)  $\sin 3 > \tan 3$   
 d)  $\sin 3 < 0$       e)  $\tan 3 > 0$
13. Which one of the following is not possible?      a)  $\tan x = 10$       b)  $\cot x = -3$       c)  $\sin x = \frac{\pi}{4}$   
 d)  $\cos x = \frac{\sqrt{2}}{100}$       e)  $\csc x = \frac{1}{2}$ .
14. Which one of the following is undefined?  
 1)  $\cos 40^\circ$       2)  $\sec 60^\circ$       3)  $\tan 360^\circ$       4)  $\cot 180^\circ$       5)  $\sin 180^\circ$
15. If  $\csc \theta > 0$ , and  $\cot \theta < 0$ , then in which quadrant will  $\theta$  terminate?
16. In the following figure, find  $3x + y - z - t$ .
17. Which one of the following is impossible:

- (a)  $\sin^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right) = \frac{1}{2}$ .
- (b)  $\sec x = e^2$ .
- (c)  $\cos \beta = \frac{3}{\pi}$ .
- (d) If  $0^\circ < \alpha < 90^\circ$ , then  $\tan \alpha > \sin \alpha$ .

18. If  $\cos \theta = -\frac{1}{\sqrt{6}}$  and  $\tan \theta = \sqrt{5}$ , then find  $\csc \theta$ .

19. If  $\cot^2 x = 4$  and  $x$  terminates in quadrant *IV*, then a)  $\cos x = 2$     b)  $\sec x = \frac{\sqrt{3}}{2}$     c)  $\csc x = -\sqrt{5}$     d)  $\tan x = \frac{1}{4}$     e)  $\sin x = 1$

20. If  $\tan x = -\frac{4}{3}$  and  $\sec x = -\frac{5}{3}$ , then find  $\csc x$ .

21. If  $\tan \alpha = m$ , and  $\alpha$  terminates in quadrant *II*, then write  $\csc x$  in terms of  $m$ .

22. If  $\cos t = \frac{2}{3}$  and  $t$  terminates in quadrant *IV*, then find  $\tan t$ .

23. If  $\sec \theta = -\frac{3}{2}$  and  $\tan \theta = \frac{\sqrt{5}}{2}$ , then find  $\csc \theta$ .

24. If  $\alpha = -\frac{2\pi}{5}$ ,  $\beta = 288^\circ$ , and  $\theta = 72^\circ$ , then which one of the following is False?

- (a)  $\tan \alpha = -\tan \beta$ .
- (b)  $\sin \alpha = -\sin \theta$ .
- (c)  $\sin^2 \alpha + \cos^2 \theta = 1$ .
- (d)  $\sec \alpha = \sec \theta$ .
- (e)  $\alpha$  and  $\beta$  are coterminal.

## 4 Section 5.4

1. If the point  $\left(-1, \frac{3}{4}\right)$  lies on the terminal side of angle  $\theta$  in standard position, then find the exact value of  $\sin(-\theta)$ .
2. Determine which quadrant contains the point  $P(16)$  that lies on the unit circle and then find the sign of  $\sin(16)$ .
3. Evaluate  $\tan\left(-\frac{34\pi}{6}\right)$ .
4. Find the exact value of  $\cos \frac{29\pi}{3} - \tan \frac{21\pi}{4} + \csc(-210^\circ)$ .

5. Find the rectangular coordinates  $(x, y)$  of the point  $P\left(-\frac{16\pi}{3}\right)$  on the unit circle.
6. Find the exact value of  $5 \cot(150^\circ) - 6 \tan(-300^\circ)$ .
7. Write  $\sec \theta$  in terms of  $\cot \theta$ , with  $\pi < \theta < \frac{3\pi}{2}$ .
8. Let  $W$  be the wrapping function, and let  $W(t) = P(x, y)$ . Find  $W\left(-\frac{45\pi}{4}\right)$ .
9. The expression  $\sec^2 x + \csc^2 x$  is identical to:
  - (a)  $\sin^2 x \cdot \cos^2 x$
  - (b)  $\sec^2 x \cdot \csc^2 x$
  - (c)  $\sec^2 x \cdot \tan^2 x$
  - (d)  $\csc^2 x \cdot \cot^2 x$
  - (e)  $\sin^2 x \cdot \tan^2 x$
10. Is the function  $f(x) = \frac{\cot x}{x}$  even, odd, or neither?
11. Write  $\frac{\tan t - \cot t}{\tan t}$  in terms of a single trigonometric function.
12. Find the values of  $x$  and  $y$  such that  $W\left(-\frac{7\pi}{3}\right) = P(x, y)$ .
13. Is the function  $f(x) = x - \sin x$  even, odd, or neither?
14. In which quadrant does the point  $P\left(6\frac{1}{3}\right)$  terminate?
15. Find the exact value of  $\cos\left(-\frac{59\pi}{4}\right)$ .
16. Find the rectangular coordinates of the point  $P\left(-\frac{31\pi}{6}\right)$  on the unit circle.
17. If  $f(x)$  is a function such that  $f\left(x + \frac{1}{2}\right) = f(x)$  for all  $x$  in the domain of  $f$ , then:
  - (a)  $f(x)$  is a periodic function of period  $\frac{1}{2}$ .
  - (b)  $f(x)$  is a periodic function of period 2.
  - (c)  $f(x)$  is a periodic function of period  $\frac{\pi}{2}$ .
  - (d)  $f(x)$  is not a periodic function.
18. Simplify the following:

- (a)  $\frac{\sin t}{1+\cos t} + \frac{1+\cos t}{\sin t}$ .
- (b)  $\frac{1}{\cos \theta} - \frac{\cos \theta}{1+\sin \theta}$ .
- (c)  $\frac{1-\cos t}{\sin t} - \frac{1}{\cot t + \csc t}$ .
- (d)  $\frac{\sin x}{\sec x - 1} + \frac{\sin x}{\sec x + 1}$ .

19. Find the value of  $\tan 945^\circ - \sin\left(-\frac{79\pi}{6}\right)$ .
20. If the point  $(-3, 4)$  lies on the terminal side of an angle  $\theta$  in standard position, then find  $\sin(-\theta) + \sec \theta$ .
21. If  $\csc \theta = -2$ , and  $\theta$  is in quadrant *III*, then find  $\cot(-\theta) + \cos(-\theta)$ .
22. Find the value of  $(\sin 510^\circ)(\csc 330^\circ) + [\cos(-330^\circ)](\sec 210^\circ)$ .
23. Find the value of  $\cos\left(-17\frac{\pi}{3}\right)$ .
24. If  $\csc(-x) = 3$ , and  $x$  terminates in quadrant *IV*, then find  $\sec x$ .
25. If  $\left(\frac{-2}{3}, \frac{\sqrt{5}}{3}\right)$  is the point on the unit circle corresponding to the arc length  $S$ , then find the point on the unit circle corresponding to the arc length  $\pi - S$ .
26. If  $P$  and  $Q$  are two points on the unit circle corresponding to the arc lengths  $\frac{23\pi}{4}$  and  $\frac{274\pi}{40}$  respectively, then find the shortest distance along the unit circle from  $P$  to  $Q$ .
27. If the point  $\left(\frac{\sqrt{3}}{6}, y\right)$ , where  $y < 0$  corresponds to an arc length  $S$  on the unit circle, then find the point on the unit circle corresponds to the arc length  $\pi - S$ .
28. Find the coordinates of the point corresponding to an arc length of  $-\frac{5\pi}{6}$  on the unit circle.
29. Find the coordinates of the point corresponding to an arc length of  $-\frac{3\pi}{2}$  on the unit circle.
30. Find the point where the terminal side of angle  $\theta = 480^\circ$  intersects the unit circle.
31. Find the point where the line segment from the origin to the point  $(-7, 24)$  intersects the unit circle.
32. On the unit circle, if the arc length  $\frac{100\pi}{3}$  terminates at  $(a, b)$  and the arc length  $\frac{55\pi}{3}$  terminates at the point  $(c, d)$ , then find  $ac + bd$ .
33. On the unit circle, if the point  $\left(\frac{3}{5}, y\right)$  corresponds to arc length  $t$  where  $y < 0$ , then find  $\cos(3\pi - t) + \sin(t - 3\pi)$ .

34. If angle  $\theta$  terminates in quadrant  $IV$ , the express  $\cos \theta$  in terms of  $\tan \theta$ .
35. If  $\sin x \cos x = \frac{3}{4}$ , then find the value of  $\frac{\tan^2 x - \sec^2 x}{\sec^2 x + \csc^2 x}$ .
36. Replace  $x$  with  $3 \sin \theta$  to write  $\sqrt{9 - x^2}$  in terms of a single trigonometric function.
37. If  $\cos \theta = \frac{3}{5}$  and  $\theta$  is in quadrant  $IV$ , then find  $\tan(-\theta)$ .
38. Simplify the following:

(a)  $\frac{\csc x + \sec x}{\tan x + \cot x}$

(b)  $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} + \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta - \cos \theta}$

(c)  $\frac{\tan x}{\sec x - \tan x} - \frac{\sec x}{\sec x + \tan x}$

(d)  $\frac{\cot x}{1 + \sin x} + \frac{\cot x}{1 - \sin x}$

(e)  $\frac{\tan^2 x - \sin^2 x}{\csc^2 x + \sec^2 x}$

(f)  $\frac{-\sin x \csc x + \cos^2 x}{\cot^2 x + 1}$

(g)  $\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta}$

(h)  $\frac{\tan^2 x + \sec^2 x + 1}{\tan^2 x}$

(i)  $\frac{\csc \theta + \cot \theta}{\tan \theta + \sin \theta}$

(j)  $\sec^4 x - 2 \sec^2 x \tan^2 x + \tan^4 x$ .

(k)  $(1 - \csc^2 x)(\sec^2 x - 1)$

(l)  $\csc^4 \theta - \cot^4 \theta$

(m)  $\frac{\tan^2 x}{1 - \cos x} - \frac{\tan^2 x}{1 + \cos x}$

39.  $\frac{\sin x}{1 + \cos x} =$

(a)  $\frac{1 - \cos x}{\sin x}$

(b)  $\frac{1 + \cos x}{\sin x}$

(c)  $\frac{\cos x}{1 + \sin x}$

(d)  $\frac{\tan x}{1 + \cot x}$

(e)  $\frac{\cot x}{1 + \tan x}$



## 5 Section 5.5

1. Find  $k$  if  $[k - 1, 3k + 4]$  is the range of a periodic function of amplitude  $\frac{7}{2}$ .
2. How many x-intercepts does the graph of  $y = \sin\left(2x - \frac{\pi}{2}\right)$ ,  $x \in \left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$  have and how many relative maximum and minimum values?
3. If  $f(x) = a \cos bx$ ,  $b > 0$  has period = 6 and  $f(3) = 4$ , then find  $f\left(\frac{21}{4}\right)$ .
4. Given  $y = -\left|3 \sin \frac{2x}{3}\right|$ 
  - (a) Find the amplitude, period, and Graph the above function
  - (b) Find the maximum and the minimum.
5. Given  $f(x) = \left|\frac{1}{3} \cos 3x\right|$ 
  - (a) Graph  $f$ .
  - (b) Find the period and the amplitude of the graph of  $f$ .
  - (c) Find the range of  $f$ .
6. Find the interval where the graph of  $y = \frac{3}{4} \cos \frac{x}{2}$ , with  $-2\pi \leq x \leq 2\pi$ , is below the x-axis.
7. Let  $a$  be a real number in  $[-1, 1]$ . Then the equation  $\sin x = a$ ,  $\frac{\pi}{6} \leq x \leq \frac{\pi}{6} + 4\pi$  has
  - (a) Exactly two solutions.
  - (b) Four solutions or two solutions.
  - (c) Two solutions or no solutions.
  - (d) Four solutions or no solutions.
8. The figure given below represents the graph of a cosine function. Find a)The amplitude.  
b)The phase shift      c)The equation of the graph.
9. Find the period of the function  $\sqrt{\sin^2(2x)}$ .
10. If  $f(x) = 4 \cos\left(\frac{1}{2}x + \frac{\pi}{4}\right)$ , then find its amplitude, phase shift and period. Also, draw the graph of  $f$  over one cycle.
11. The adjacent figure represents, over 1 period, the following function:

- (a)  $\cos\left(x + \frac{\pi}{4}\right)$ .
- (b)  $\sin\left(x + \frac{\pi}{4}\right)$ .
- (c)  $\cos\left(2x + \frac{\pi}{2}\right)$ .
- (d)  $\sin\left(2x + \frac{\pi}{2}\right)$ .
- (e)  $-\sin\left(2x + \frac{\pi}{2}\right)$ .

12. Find the period and the amplitude of the function  $y = -\frac{1}{2} \cos 4x$ .

13. Given the function  $f(x) = -2 \sin\left(2x - \frac{\pi}{4}\right)$ .

- (a) Find the period of  $f(x)$ .
- (b) Find the phase shift of the graph of  $f(x)$ .
- (c) Find the range of  $f(x)$ .
- (d) Sketch the graph of  $f(x)$  over the interval  $\left[\frac{\pi}{8}, \frac{9\pi}{8}\right]$ .

14. Graph the function  $y = 3 \sin\left(2x - \frac{\pi}{2}\right)$  over one period.

15. Find the amplitude, phase shift, and the period of the function  $y = -\frac{3}{4} \cos\left(\frac{\pi}{4} - 2x\right)$ .

16. Find the period of the function  $y = \sin^2(2\pi x)$ .

17. Find the amplitude, phase shift, and the period of the function  $y = -6 \sin(\pi - 4x)$ .

18. If the adjacent figure represents the graph of  $y = a \sin b(x + c)$ ,  $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ , then find the values of  $a$ ,  $b$ , and  $c$ .

19. Find the amplitude, phase shift, and the period of the function  $y = \frac{1}{2} - \cos\left(2x - \frac{\pi}{2}\right)$ .

20. The function  $y = 2 + \sin\left(x - \frac{\pi}{3}\right)$  is

- (a) increasing on  $[0, \pi]$ .
- (b) decreasing on  $[0, \pi]$ .
- (c) decreasing on  $\left[0, \frac{\pi}{2}\right]$ .
- (d) increasing on  $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ .

(e) decreasing on  $\left[\frac{\pi}{3}, \frac{5\pi}{6}\right]$ .

21. If  $f(x) = a \cos\left(bx - \frac{\pi}{3}\right)$ , ( $a > 0$ ) has Amplitude = 2 and Phase shift =  $\frac{\pi}{2}$ , then find  $f\left(\frac{\pi}{2}\right)$ .

22. A cosine function of Amplitude = 3 and Period = 2 is

(a)  $y = 3 \cos x$

(b)  $y = -3 \cos \pi x$

(c)  $y = 2 \cos \frac{2\pi}{3}x$

(d)  $y = -2 \cos 3x$

(e)  $y = 3 \cos 2\pi x$

23. Find the intervals in which the function  $y = \sin\left(2x - \frac{\pi}{2}\right)$ ,  $0 \leq x \leq \frac{5\pi}{4}$  is decreasing.

24. Let  $f(x) = a \cos bx$  with Period = 10. Suppose  $f(5) = 2$ , then find  $f(25)$ .

25. Which one of the following is FALSE?

(a)  $\cot 4 > \cot 6$ .

(b)  $\tan 6 > \tan 4$ .

(c) The angle  $701^\circ$  is coterminal with the angle  $341^\circ$ .

(d)  $f(x) = \csc x \tan x$  is an even function.

(e)  $f(x) = |\sin 2x|$  has period of  $\frac{\pi}{2}$ .

## 6 Section 5.6

1. How many vertical asymptotes does the graph of  $y = 3 \tan\left(\frac{1}{3}x - \frac{\pi}{6}\right)$ , for  $-6\pi \leq x \leq 6\pi$  have?
2. Given  $f(x) = 3 \cot\left(2x + \frac{\pi}{3}\right)$ . Find a)Period    b)Phase shift    c)Range    d)Sketch the graph over one full period.
3. Sketch the graph of  $y = 5 \cot \frac{2x}{3}$  on the interval  $[0, 3\pi]$  showing the period, the x-intercepts and the asymptotes.
4. Find the period, phase shift and the vertical translation of the graph of the function  $y = 4 + 2 \cot(3x - 2\pi)$ .

5. For the function  $y = \frac{3}{2} \cot\left(2x + \frac{\pi}{4}\right)$ .
- Find the period and the phase shift of the graph.
  - Sketch one full period of the graph. (Show at least three points and two asymptotes on the graph of the function).
6. Find the range and the period of the function  $f(x) = 2 + \frac{3}{2} \csc\left(\frac{x}{3} - \frac{\pi}{2}\right)$ .
7. If the graph of the function  $f(x) = 2 \cot(bx + c)$  has period  $\frac{2\pi}{3}$  and phase shift  $\frac{\pi}{2}$  to the left, then find  $f(\pi)$ .
8. If  $x = a$  and  $x = b$  are the 2 asymptotes of  $y = 4 \tan\left(\frac{1}{3}x - \frac{\pi}{6}\right)$  in the interval  $\left(-\frac{3\pi}{2}, \frac{5\pi}{2}\right)$ , then find  $a$  and  $b$ .
9. Find the range and the period of  $f(x) = -\frac{3}{2} \csc\left(3x + \frac{\pi}{2}\right) + 1$ .
10. Which of the following functions has the graph given below on the interval  $\left[\frac{\pi}{4}, \frac{7\pi}{12}\right]$  :
- $y = \frac{3}{2} \cot\left(2x + \frac{\pi}{4}\right)$ .
  - $y = \frac{-3}{2} \cot\left(3x + \frac{\pi}{4}\right)$ .
  - $y = \frac{3}{2} \cot\left(3x + \frac{\pi}{4}\right)$ .
  - $y = \frac{-3}{2} \cot\left(2x + \frac{\pi}{4}\right)$ .
  - $y = \frac{3}{2} \cot\left(3x + \frac{5\pi}{12}\right)$ .
11. The graph of the function  $y = -7 \sec \frac{1}{2}\pi x$  in the interval  $(1, 3)$  will:
- increase on  $(1, 2)$ , and decrease on  $(2, 3)$ .
  - increase on  $(1, 3)$ .
  - decrease on  $(1, 3)$ .
  - decrease on  $(1, 2)$ , and increase on  $(2, 3)$ .
  - increase on  $\left(1, \frac{3}{2}\right)$ , and decrease on  $\left(\frac{3}{2}, 3\right)$ .
12. Let  $f(x) = 2 \tan\left(\frac{x}{2} - \frac{\pi}{2}\right)$ , over  $(-2\pi, 2\pi)$ , find

- (a) x-intercepts of  $f$  over the given interval.
  - (b) y-intercept of  $f$  over the given interval.
  - (c) all vertical asymptotes over the given interval.
  - (d) the range of  $f$ .
  - (e) Using parts(a), (b), (c) and (d), sketch the graph of  $f(x)$  over the given interval.
13. Graph the function  $y = 1 - 3 \tan(2x - \pi)$  over one period.
14. Find the range of the function  $y = 3 \sec x - 1$ .
15. Find the vertical asymptotes of  $y = 3 \cot\left(x - \frac{\pi}{2}\right)$ .
16. The adjacent figure represents the graph of: a)  $y = 3 \csc x$     b)  $y = -\sec x$     c)  $y = 2 \csc x + 1$   
d)  $y = 2 \sec x + 1$     e)  $y = \csc x + 2$
17. Find the period and the range of the function  $f(x) = 1 + \sec \frac{x}{3}$ .
18. Find the range of the function  $y = 2 \csc x - 1$ .
19. Find the period of  $y = \tan^2 x$ .
20. Find the domain of  $y = \tan x$ .
21. The adjacent figure represents the graph of:
- (a)  $y = -\cot\left(\frac{1}{2}x - \frac{\pi}{8}\right)$ .
  - (b)  $y = \cot\left(\frac{1}{2}x - \frac{\pi}{8}\right)$ .
  - (c)  $y = -\cot\left(x - \frac{\pi}{4}\right)$ .
  - (d)  $y = -\cot\left(x + \frac{\pi}{4}\right)$ .
  - (e)  $y = \cot\left(x + \frac{\pi}{4}\right)$ .
22. If the asymptotes of  $f(x) = 3 - 2 \sec\left(2x - \frac{\pi}{4}\right)$  are given by  $x = \frac{A+Bn}{C}\pi$ , where  $n$  is an integer, then find the value of  $A$ ,  $B$ , and  $C$ .
23. Which one of the following has Period  $2\pi$  and Phase shift  $\frac{\pi}{6}$  to the right?
- (a)  $y = \sec\left(x + \frac{\pi}{6}\right)$ .

(b)  $y = \sec(6x - \pi)$ .

(c)  $y = \sec(x - 3\pi)$ .

(d)  $y = \frac{1}{2} \tan \frac{1}{2}(x + 3\pi)$ .

(e)  $y = \csc\left(x - \frac{\pi}{6}\right)$ .

24. Find the range of the function  $f(x) = -\frac{5}{4} + \frac{3}{2} \csc\left(2x - \frac{\pi}{6}\right)$ .

25. Find the number of vertical asymptotes of  $y = 2 \cot\left(3x - \frac{\pi}{2}\right)$  in the interval  $\left(-\frac{\pi}{4}, \pi\right)$ .

26. Find the set of all x-intercepts of  $y = \cot 2x$ ,  $0 \leq x \leq 2\pi$ .

27. The adjacent figure represents a)  $\csc x$     b)  $\sec x$     c)  $\tan x$     d)  $\cot x$     e) none of the above.

28. Find the vertical asymptotes of  $y = \cot\left(2x + \frac{\pi}{3}\right)$ .

## 7 Section 5.7

1. The function  $y = x + \sin x$  is a) not periodic    b) has period  $2\pi$     c) has period  $1 + \pi$     d) has period  $1 - \pi$     e) has period  $\pi$ .

2. Find the x-intercepts of the graph of  $y = x \csc x$ .

3. How many zeros does the function  $f(x) = \sec \frac{\pi}{2}x - x$  have in the interval  $[0, 4]$ ? ( Hint: Use graphs)