King Fahd University of Petroleum and Minerals
College of Sciences
Prep-Year Math Program

Key

Math 001 Exam I
Term 021 (2002-2003)
Saturday, October 19, 2002
Time Allowed: 90 Minutes

Graders

Student's Name: _____

ID #: _____ **Section #:** _____

This exam consists of Two parts

Part I : Multiple Choice Bubble the correct answer on the OMR sheet.
Part II : Written Questions Provide neat and complete solutions.
Show all necessary steps for full credit.

Calculators, Pagers, or Mobiles are NOT allowed during this exam.

Question	Points	Grader
Part I: (1 - 8)	12	
Part II:		
1	3	
2	2	
3	3	
4	4	
5	4	
6 (a)	2	
6 (b)	2	
7	3	
8	5	
9	4	
Total		
	44	

Part I: (12-points) Multiple Choice Questions (MCQ).
Encircle The Correct Answer

1. Which one of the following statements is TRUE?

   (a) The product of two composite numbers is a composite number.

   (b) The sum of two composite numbers is a composite number.

   (c) The sum of two irrational numbers is an irrational number.

   (d) The product of two irrational numbers is an irrational number.

2. The distance between the two points whose coordinates on a number line are $-\pi$ and 3, is equal to

   (a) $\pi + 3$

   (b) $|-\pi + 3|$

   (c) $|\pi - 3|$

   (d) $-(-\pi + 3)$

3. Which one of the following statements is TRUE for any real number x?

   (a) $\sqrt[3]{-x^3} = -x$

   (b) $\sqrt{16x^2} = 4x$

   (c) $\sqrt{(-2x)^2} = -2x$

   (d) $\sqrt[3]{64x^3} = 4|x|$

4. If $i = \sqrt{-1}$, then $i^{50} + i^{51} + i^{52} =$

   (a) $-i$

   (b) i

   (c) -1

   (d) 0

5. The expression $\frac{x+y}{x-y} \cdot \frac{x^{-1}-y^{-1}}{x^{-1}+y^{-1}}$ is equal to

(a) -1

(b) 0

(c) $\frac{0}{0}$

(d) $x^2 - y^2$

6. If $A = \frac{1}{2}(B+x)y$ and $y \neq 0$, then $B =$

(a) $\frac{2A - xy}{y}$

(b) $\frac{A + 2xy}{y}$

(c) $\frac{2A - x}{y}$

(d) $2A - \frac{1}{2}xy$

7. The coefficient of xy^2 in the expression$(3x-2y)^3$ is equal to

(a) 36

(b) -36

(c) 18

(d) -12

8. Which one of the following statements is FALSE?

(a) The equation $\frac{x^2-4}{x-2} = 4$ has a real solution.

(b) If $x \neq 3$, then $\frac{x^3-27}{x-3} = x^3 + 3x + 9$ is an identity.

(c) The equation $|3x-5| = -8$ is a contradiction.

(d) The equation $5x + 7 = 3$ is a conditional equation.

Part II: Written Questions.
[Provide neat and complete solution. Show necessary steps for full credit.]

1. (3-points) Given the sets

$$A = \{z | z = -|x| + x, \text{ where } x \text{ is an integer with } -4 < x \leq 0\},$$
$$\text{and} \quad B = \{z | z = 2x - 2, \text{ where } x \text{ is an integer with } -3 \leq x < 0\}.$$

List the elements of the sets A, B, and $A \cap B$.

$A = \{-6, -4, -2, 0\}$ --- 1 Point.

$B = \{-8, -6, -4\}$ --- 1 Point

$A \cap B = \{-6, -4\}$ --- 1 Point

2. (2-points) Given the inequality $x \leq -5$ or $2 < x \leq 6$.

(a) Graph the given inequality on a number line:

--- 1 Point

(b) Write the given inequality using interval notation.

$(-\infty, -5] \cup (2, 6]$ --- 1 Point

3. (3 points) Simplify $(3x - 5)(2x^2 + 4x - 6)$. Write the result in standard form.

$(3x-5)(2x^2+4x-6)$

$= 3x(2x^2+4x-6) - 5(2x^2+4x-6)$ --- 1 Point

$= 6x^3 + 12x^2 - 18x - 10x^2 - 20x + 30$ --- 1 Point

$= 6x^3 + 2x^2 - 38x + 30.$ --- 1 Point

4. (4-points) Given that $0 < x < \frac{1}{8}$, write the expression $\left|\frac{|x-\frac{1}{4}|}{|x-\frac{1}{8}|+|x+\frac{1}{8}|}\right|$ without absolute value symbols and in the simplest form.

The given expression $= \dfrac{|x-\frac{1}{4}|}{|x-\frac{1}{8}|+|x+\frac{1}{8}|}$ --- 1 Point

$= \dfrac{-(x-\frac{1}{4})}{-(x-\frac{1}{8})+(x+\frac{1}{8})}$ --- 1 Point

$= \dfrac{-x+\frac{1}{4}}{-x+\frac{1}{8}+x+\frac{1}{8}}$ --- 1 Point

$= \dfrac{-x+\frac{1}{4}}{\frac{1}{4}} = -4x+1$ --- 1 Point

5. (4-points) Simplify $\left[\frac{(-2y)^0 y^{-1}(2y)^3}{(2y^{-2})^{-1}y^{-4}}\right]^{-1/2}$, where $y \neq 0$. Write the result using positive exponents only.

The given expression $= \left[\dfrac{(1)(y^{-1})(8)(y^3)}{2^{-1}(y^2)(y^{-4})}\right]^{-1/2}$ --- 1 Point

$= \left[16\, y^4\right]^{-1/2}$ --- 1 Point

$= 4^{-1}\, y^{-2}$ --- 1 Point

$= \dfrac{1}{4y^2}$. --- 1 Point

6. (a) (2-points) Factor $9x^2 - 24xy + 16y^2 - 100z^2$.

The given expression $= (3x-4y)^2 - (10z)^2$ ... 1 point)

$= (3x-4y-10z)(3x-4y+10z)$ ... 1 point

(b) (2-points) Rationalize the numerator $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$. Write your answer in the simplest form.

$\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}+\sqrt{3})^2}$ ... 1 point

$= \frac{2-3}{2+2\sqrt{6}+3} = \frac{-1}{5+2\sqrt{6}}$ . ... 1 point

7. (3-points) Simplify $-3x\sqrt[3]{54x^4} + 2\sqrt[3]{16x^7}$. Write the result in the simplest form.

The given expression $= -3x\sqrt[3]{3^3 \cdot 2 \cdot x^3 \cdot x} + 2\sqrt[3]{2^3 \cdot 2 \cdot x^6 \cdot x}$

... 1 point

$= -9x^2\sqrt[3]{2x} + 4x^2\sqrt[3]{2x}$

... 1 point

$= -5x^2\sqrt[3]{2x}$ ... 1 point.

8. (5-points) Simplify $\frac{x}{2x-1} - \frac{1}{2x^2-7x-4} \div \frac{x+3}{x^2-x-12}$. Write the result in the simplest form.

The given expression $= \frac{x}{2x-1} - \frac{1}{(2x+1)(x-4)} \cdot \frac{(x-4)(x+3)}{(x+3)}$ ... 2 points

$= \frac{x}{2x-1} - \frac{1}{2x+1}$ ... 1 point

$= \frac{2x^2+x-2x+1}{(2x-1)(2x+1)}$ ... 1 point

$= \frac{2x^2-x+1}{(2x-1)(2x+1)}$ ... 1 point

(The answer $\frac{2x^2-x+1}{4x^2-1}$ is also accepted)

9. (4-points) Write the conjugate of the complex number $\frac{1}{(2+i)^2-8i}$ in standard form.

$\frac{1}{(2+i)^2-8i} = \frac{1}{4+4i-1-8i} = \frac{1}{3-4i}$ ... 1 point

$= \frac{3+4i}{(3-4i)(3+4i)}$ ... 1 point

$= \frac{3+4i}{9+16} = \frac{3}{25} + \frac{4}{25}i$ ... 1 point

The conjugate of $\frac{3}{25} + \frac{4}{25}i = \frac{3}{25} - \frac{4}{25}i$ ... 1 point