King Fahd University of Petroleum and Minerals
College of Sciences
Prep-Year Math Program

KEY

Math 001 Exam II
Term 021 (2002-2003)
Saturday, December 21, 2002
Time Allowed: 90 Minutes

KEY

Student's Name: _____

ID #: _____ **Section #:** _____

This exam consists of Two parts

Part I : Multiple Choice: Bubble the correct answer on the OMR sheet.
Part II : Written Questions: Provide neat and complete solutions.
Show all necessary steps for full credit.

Calculators, Pagers, or Mobiles are NOT allowed during this exam.

Question	Points	GRADER
Part I: MCQ (1 - 6)	12	
Part II: Written		
1	5	
2	4	
3	4	
4	4	
5	5	
6	4	
7	6	

Total	
	44

Part I: (12-points) Multiple Choice Questions (MCQ).
Bubble the Correct Answer in the OMR Sheet.

1. Let k be a nonzero real constant, then the equation $2x^2 + 2kx + 5k^2 = 0$

(a) has two distinct complex roots that are not real

(b) has a real root that is a double root

(c) has two distinct real roots

(d) has more than two roots

2. The solution set of the inequality $\frac{(x+1)^3(2-x)}{x^2} \leq 0$ is equal to

(a) $(-\infty, -1] \cup [2, \infty)$

(b) $[-1, 0) \cup (0, 2]$

(c) $(-\infty, 0) \cup [2, \infty)$

(d) $[2, \infty)$

3. The graph of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$ is

(a) tangent to the x-axis only

(b) tangent to the y-axis only

(c) tangent to both the axes

(d) not tangent to any of the axes

4. If $-3 \leq x \leq 0$, then the range of the function $f(x) = (x+1)^2 + 1$ is equal to

(a) $[1, 5]$

(b) $[2, 5]$

(c) $[1, \infty)$

(d) $[5, \infty)$

5. Identify the set of ordered pairs (x, y) or the equation that defines y as a function of x.

(a) $\{(2, 3), (3, 2), (4, 5), (5, 4)\}$

(b) $|y| = x - 5$

(c) $\{(2, 3), (2, 4), (2, 5), (2, 6)\}$

(d) $|x| + |y| = 10$

6. If $\left(\frac{7}{4}, \frac{11}{4}\right)$ is the midpoint of a line segment with endpoints (x, y) and $\left(-\frac{1}{2}, \frac{5}{3}\right)$, then the value of x is equal to

(a) 4

(b) -4

(c) $\frac{9}{4}$

(d) $-\frac{5}{8}$

Part II: Written Questions.
[Provide neat and complete solution. Show all necessary steps for full credit.]

1. (5-points) Solve $\sqrt{2x} = \sqrt{x+7} - 1$.

$\Rightarrow \quad 2x = x+7 - 2\sqrt{x+7} + 1$

$\Rightarrow \quad x - 8 = -2\sqrt{x+7}$ ... 1 point

$\Rightarrow \quad x^2 - 16x + 64 = 4x + 28$

$\Rightarrow \quad x^2 - 20x + 36 = 0$ ... 1 point

$\Rightarrow \quad (x-2)(x-18) = 0$

$\Rightarrow \quad x = 2 \quad \text{or} \quad x = 18$ ... 1 point

Check: for $x=2 \Rightarrow \sqrt{4} = \sqrt{9} - 1$ ✓
for $x=18 \Rightarrow \sqrt{36} = \sqrt{25} - 1$ ✗ } ... 1 point

$\Rightarrow$ The only solution is 2 ... 1 point

2. (4-points) Solve the inequality $0 < |3 - 2x| \leq 7$. Give the solution set using interval notation.

$\Rightarrow \quad x \neq \frac{3}{2} \quad \text{and} \quad -7 \leq 3 - 2x \leq 7$ ... 1 point

$\Rightarrow \quad x \neq \frac{3}{2} \quad \text{and} \quad -10 \leq -2x \leq 4$ ... 1 point

$\Rightarrow \quad x \neq 3/2 \quad \text{and} \quad -2 \leq x \leq 5$ ... 1 point

$\Rightarrow$ The solution set $= [-2, \frac{3}{2}) \cup (3/2, 5]$ ... 1 point

3. (4-points) Find the domain of the function $f(x) = \sqrt{4x^3 - 3x^2 - x}$. Write your answer using interval notation.

We must have $4x^3 - 3x^2 - x \geq 0$ ... 1 point

$\Rightarrow x(4x^2 - 3x - 1) \geq 0$

$\Rightarrow x(4x+1)(x-1) \geq 0$ ... 1 point

$\Rightarrow$ The sign diagram

	$-\infty$		$-\frac{1}{4}$		0		1		$+\infty$
$4x+1$	$-$		0	$+$	$+$	$+$	$+$	$+$	$+$
x	$-$		$-$	$-$	0	$+$	$+$	$+$	$+$
$x-1$	$-$		$-$	$-$	$-$	$-$	0	$+$	$+$
	$-$		0	$+$	0	$-$	0	$+$	

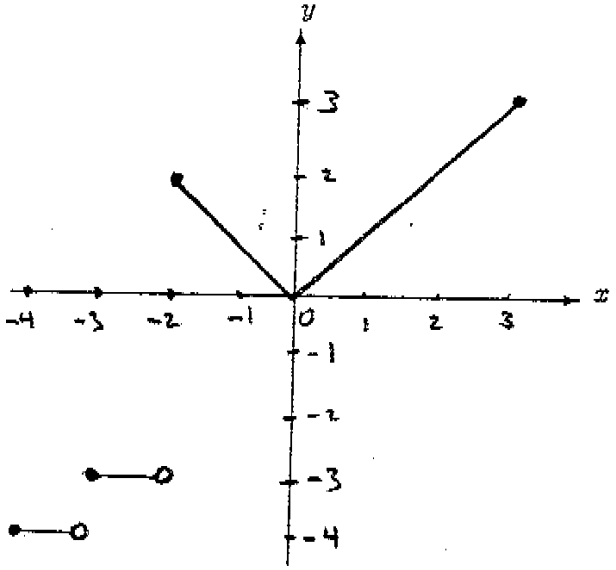
... 1 point

$\Rightarrow$ The domain $= [-\frac{1}{4}, 0] \cup [1, \infty)$ ... 1 point

4. (4-points) Given the function $f(x) = \begin{cases} [\![x]\!], & -4 \leq x < -2 \\ |x|, & -2 \leq x \leq 3 \end{cases}$, where $[\![x]\!]$ is the greatest integer less than or equal to x.

(a) Sketch the graph of $f(x)$.

x	-4	-3	-2	-1	3
y	-4	-3	2	1	3



3 ... points

(b) Evaluate $f\left(-\frac{5}{2}\right) + f\left(\frac{5}{2}\right)$.

$= [\![-\frac{5}{2}]\!] + |\frac{5}{2}|$

$= -3 + \frac{5}{2} = -\frac{1}{2}$ ... 1 point

5. (5-points) Let L be the line with x-intercept $(k, 0)$, and y-intercept $(0, 3k^2)$ where $k \neq 0$. If L is perpendicular to the line $2x + 3y = 5$, then:

(a) Find the value of k.

The slope of $L = \frac{3k^2 - 0}{0 - k} = -3k$ ... 1 point

The slope of the given line $= -\frac{2}{3}$ ... 1 point

$\Rightarrow (-3k)(-\frac{2}{3}) = -1$

$\Rightarrow k = -\frac{1}{2}$ ... 1 point

(b) Use part (a) to find the equation of L. Write the equation in the form $y = mx + b$.

From part (a) $\Rightarrow$

The slope $m = -3k = \frac{3}{2}$

The y-intercept $(0, b) = (0, 3k^2) = (0, \frac{3}{4})$ } ... 1 point

$\Rightarrow$ The required equation is

$y = \frac{3}{2}x + \frac{3}{4}$ ... 1 point

6. (4-points) The sum of a real number and twice its reciprocal is $\frac{73}{6}$. Find all such numbers.

If x is a such number $\Rightarrow$

$x + \frac{2}{x} = \frac{73}{6}$ ... 1 point

$\Rightarrow 6x^2 - 73x + 12 = 0$ ... 1 point

$\Rightarrow (6x - 1)(x - 12) = 0$ ... 1 point

$\Rightarrow$ The numbers are $\frac{1}{6}$ and 12 ... 1 point

7. (6-points) Encircle TRUE or FALSE for each of the following statements:

(a) The maximum value of the function $f(x) = -2x^2 + 8x + 3$ is 11. TRUE or FALSE

$f(x)$ is maximum at $x = -\frac{8}{-4} = 2$

$\Rightarrow f(2) = -8 + 16 + 3 = 11$ ✓

(b) The image of the point $P(5, -3)$ with respect to the origin is the point $Q(3, -5)$. TRUE or FALSE

The image is $(-5, 3)$ ×

(c) The function $f(x) = x^3|x - 1|$ is an odd function. TRUE or FALSE

$f(-x) = (-x)^3|-x-1| = -x^3|x+1| \neq -f(x)$

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(d) The graph of the quadratic function $f(x) = 3(x - 1)^2 + 5$ is symmetric with respect to the vertical line $x = 1$.

TRUE or FALSE

$x=1$ is the axis of the symmetry of the given parabola

✓

(e) The distance between the points $P(1, 5)$ and $Q(4, 9)$ is equal to 7. TRUE or FALSE

The distance $= \sqrt{(4-1)^2 + (9-5)^2} = \sqrt{9+16} = 5 \neq 7$

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(f) The graph of $y = (-x - 2)^2$ is the graph of $y = x^2$ shifted right horizontally 2 units. TRUE or FALSE

$y = (-x-2)^2 \Rightarrow y = (x+2)^2$

$\Rightarrow$ the shift is to left.

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