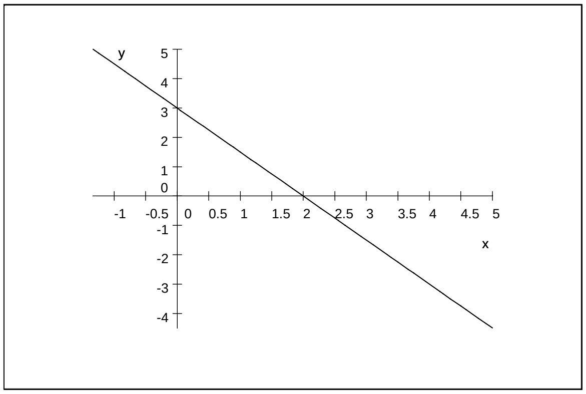
1. 1. $r\left(3\cos\theta + 2\sin\theta\right) = 6 \Longrightarrow$
   $3r\cos\theta + 2r\sin\theta = 6 \Longrightarrow$
   $\boxed{3x + 2y = 6}$
   Graph:



   2. $x^2 + y^2 = 4x \Longrightarrow$
   $r^2 = 4r\cos\theta \Longrightarrow$
   $\boxed{r = 4\cos\theta}$

2. 1. $r = 4 - 3\sin\frac{5\pi}{6} = \frac{5}{2}$, Therefore, $x = r\cos\theta = \frac{5}{2}\cos\frac{5\pi}{6} = -\frac{5}{4}\sqrt{3}$, $y = r\sin\theta = \frac{5}{2}\sin\frac{5\pi}{6} = \frac{5}{4}$. The point is $\boxed{\left(-\frac{5}{4}\sqrt{3}, \frac{5}{4}\right)}$.
   2. $\dfrac{dy}{dx} = \dfrac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \dfrac{-3\cos\frac{5\pi}{6}\sin\frac{5\pi}{6} + \frac{5}{2}\cos\frac{5\pi}{6}}{-3\cos\frac{5\pi}{6}\cos\frac{5\pi}{6} - \frac{5}{2}\sin\frac{5\pi}{6}} = \boxed{\frac{1}{7}\sqrt{3}}$
   3. We use the slope-point form of the equation of the straight line to get

$$y - \frac{5}{4} = \frac{1}{7}\sqrt{3}\left(x + \frac{5}{4}\sqrt{3}\right)$$

   Simplify to get: $\boxed{y = \frac{1}{7}x\sqrt{3} + \frac{25}{14}}$
   4. For horizontal tangents we must have: $r'\sin\theta + r\cos\theta = 0$. Substituting the expressions for r, r' gives $-3\cos\theta\sin\theta + (4 - 3\sin\theta)\cos\theta = 0$, which simplifies to $4\cos\theta - 6\sin\theta\cos\theta = 0$. Factor to get $\cos\theta\,(2 - 3\sin\theta) = 0$. So $\cos\theta = 0$ or $\sin\theta = \frac{2}{3}$. Solving for $0 \le \theta \le 2\pi$ gives the set of solutions $\boxed{\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \sin^{-1}\frac{2}{3}, \pi - \sin^{-1}\frac{2}{3}\right\}}$.

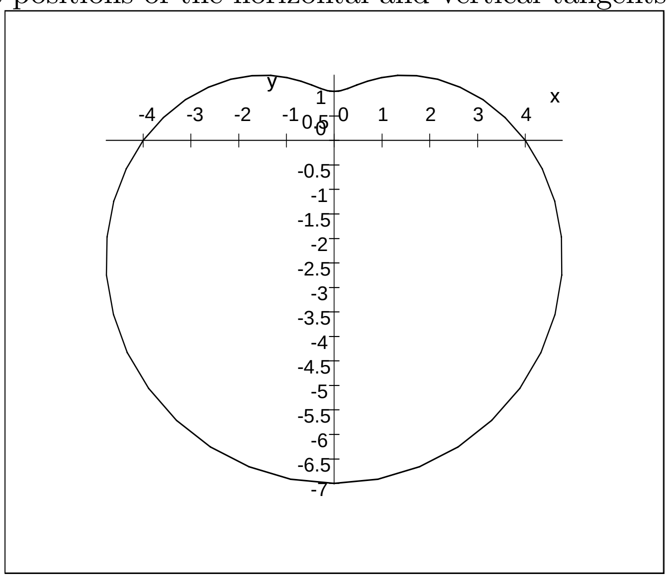
5. For vertical tangents we must have: $r' \cos\theta - r\sin\theta = 0$. Substituting the expressions for r, r' gives $-3\cos^2\theta - (4 - 3\sin\theta)\sin\theta = 0$, which simplifies to $3\sin^2\theta - 4\sin\theta - 3 = 0$. From the quadratic formula

$$\sin\theta = \frac{4 \pm \sqrt{82}}{12}.$$

The acceptable solution is

$$\sin\theta = \frac{4 - \sqrt{82}}{12},$$

the other one being greater than one. Solving for $0 \le \theta \le 2\pi$ gives the set of solutions $\boxed{\left\{\sin^{-1}\frac{4-\sqrt{82}}{12}, \pi - \sin^{-1}\frac{4-\sqrt{82}}{12}\right\}}$. The graph is shown below. Note the positions of the horizontal and vertical tangents



3. 1. Solve the two equatuions simultanuously:

$$\begin{aligned} 1 + \cos\theta &= 3\cos\theta \\ \cos\theta &= \frac{1}{2} \\ \theta &= -\frac{\pi}{3}, \frac{\pi}{3} \\ r &= \frac{3}{2} \end{aligned}$$

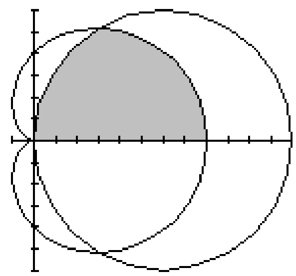
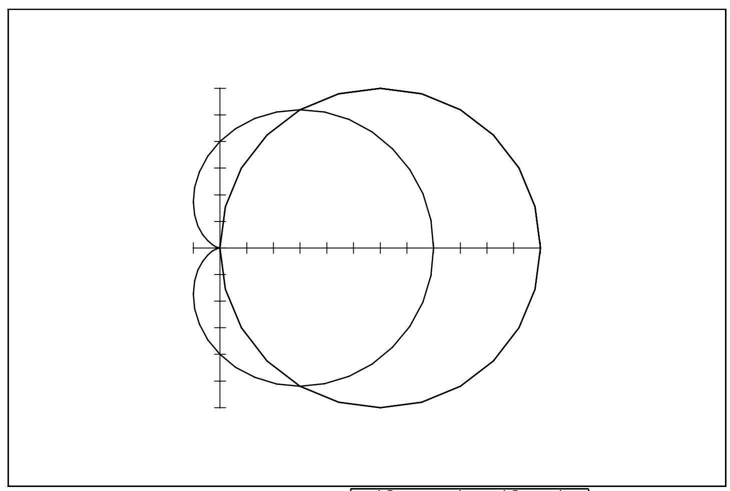


Figure 1:



The algebraic points of intersection are $\boxed{\left(\frac{3}{2}, -\frac{\pi}{3}\right), \left(\frac{3}{2}, \frac{\pi}{3}\right)}$ and from the graph we get a third point $\boxed{(0,0)}$.

2. The required area is twice the shaded one. which consists of the two partsplus

$$\begin{aligned} A &= 2\left\{\int_0^{\frac{\pi}{3}} \frac{1}{2}(1+\cos\theta)^2\, d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{9}{2}\cos^2\theta d\theta\right\} \\ &= \boxed{\int_0^{\frac{\pi}{3}} (1+\cos\theta)^2\, d\theta + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta d\theta} \end{aligned}$$

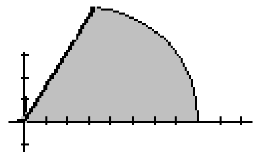


Figure 2:

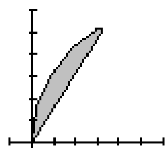


Figure 3:

4. 1. $(\mathbf{u}+2\mathbf{v})\cdot(3\mathbf{u}-\mathbf{v}) = (\langle 1,2,-3\rangle + \langle -4,0,10\rangle)\cdot(\langle 3,6,-6\rangle - \langle -2,0,5\rangle) = \langle -3,2,7\rangle \cdot \langle 5,6,-14\rangle = \boxed{-101}$

   2. $\mathbf{u}+\mathbf{v} = \langle -1,2,2\rangle$, $\|\mathbf{u}+\mathbf{v}\| = 3$. The required vector is $\frac{4}{3}\langle -1,2,2\rangle = \boxed{\langle -\frac{4}{3}, \frac{8}{3}, \frac{8}{3}\rangle}$

   3. $\text{Proj}_{\mathbf{u}}\mathbf{v} = \dfrac{\mathbf{v}\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u} = \dfrac{-17}{14}\langle 1,2,-3\rangle = \boxed{\left\langle \dfrac{-17}{14}, \dfrac{-17}{7}, \dfrac{51}{14}\right\rangle}$

   4. A vector perpindicular to $\mathbf{u}$ is $\langle 3,0,1\rangle$. A unit vector is $\boxed{\left\langle \dfrac{3}{\sqrt{10}}, 0, \dfrac{1}{\sqrt{10}}\right\rangle}$

5. 1. $|AB|^2 = 4+36+9 = 49$, $|AC|^2 = 36+16+144 = 196$, $|BC|^2 = 16+4+225 = 245$.
   $\boxed{|AB|^2 + |AC|^2 = 49+196 = 245 = |BC|^2}$. Therefore, ABC is a right triangle.

   2. $\boxed{\text{Vertex } A \text{ is } 90^\circ}$.

   3. Area of the triangle $= \dfrac{1}{2}|AB|\cdot|AC| = \dfrac{1}{2}\cdot 7\cdot 14 = \boxed{49}$