

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
MATHEMATICAL SCIENCES DEPARTMENT  
MATH 201  
EXAM # 1  
TUESDAY OCT 7, 2003

NAME:

SEC#:

ID#:

**SHOW ALL YOUR WORK**

1. (4points each)

(a) Transform the equation

$$r = \frac{6}{3 \cos \theta + 2 \sin \theta}$$

to rectangular coordinates and carefully sketch the resulting curve.

(b) Express the equation  $x^2 + y^2 = 4x$  in polar coordinates. Give your answer in the form  $r = f(\theta)$ .

2. For the curve  $r = 4 - 3 \sin \theta$  :

(a) (2points) Give the  $(x, y)$  coordinates of the point on the graph that corresponds to  $\theta = \frac{5\pi}{6}$ .

(b) (2points) Show that the slope of the tangent to the graph at  $\theta = \frac{5\pi}{6}$  is  $\frac{\sqrt{3}}{7}$ .

(c) (2points) Find the Cartesian equation of the tangent to the graph at  $\theta = \frac{5\pi}{6}$ .

(d) (1point) Give the exact values of  $\theta$  at which the graph of the curve has a horizontal tangent.

(e) (1point) Give the exact values of  $\theta$  at which the graph of the curve has a vertical tangent.

3. Consider the curves  $r = 1 + \cos \theta, r = 3 \cos \theta$ .

(a) (3points) Find all points of intersection of the two curves.

(b) (5points) Set up an integral to calculate the common area between the two curves. Do not carry out the integration.

4. Let  $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ ,  $\mathbf{v} = \langle -2, 0, 5 \rangle$ . Find:

- (a) **(2points)**  $(\mathbf{u}+2\mathbf{v}) \cdot (3\mathbf{u} - \mathbf{v})$ ,
  - (b) **(2points)** a vector of norm 4 in the same direction as  $\mathbf{u} + \mathbf{v}$ ,
  - (c) **(2points)**  $\text{Proj}_{\mathbf{u}}\mathbf{v}$ ,
  - (d) **(2points)** a unit vector perpendicular to  $\mathbf{u}$ .
5. (a) **(4points)** Show that  $A(2, 1, 6)$ ,  $B(4, 7, 9)$ ,  $C(8, 5, -6)$  are vertices of a right triangle.
- (b) **(2points)** Which vertex is  $90^\circ$ ?
- (c) **(2points)** Find the area of the triangle.