

1. The midpoint Q has coordinates $(\frac{-2+2}{2}, \frac{1+2}{2}, \frac{5+2}{2}) = (0, \frac{3}{2}, \frac{7}{2})$. The direction of the line $\mathbf{v} = \overrightarrow{PQ} = \langle 1, \frac{7}{2}, -\frac{1}{2} \rangle$. The desired parametric equations are $x = -1 + t$, $y = -2 + \frac{7}{2}t$, $z = 4 - \frac{1}{2}t$
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2. 1. The directions $\mathbf{v}_1, \mathbf{v}_2$ of L_1, L_2 are $\mathbf{v}_1 = \langle 7, 1, -3 \rangle$ and $\mathbf{v}_2 = \langle -1, 0, 2 \rangle$. $\mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} 7 & 1 & -3 \\ -1 & 0 & 2 \end{vmatrix} = \langle 2, -11, 1 \rangle \neq \mathbf{0}$. This means that $\mathbf{v}_1, \mathbf{v}_2$ are not parallel vectors. Hence L_1, L_2 are not parallel.
2. No, because the distance between the two lines is not zero as calculated in part c. below.
3. The plane Π through L_1 that is parallel to L_2 normal direction $\mathbf{v}_1 \times \mathbf{v}_2 = \langle 2, -11, 1 \rangle$ and passes through the point $(1, 3, 5)$ on L_1 . Hence it has equation $2(x - 1) - 11(y - 3) + (z - 5) = 0$. The distance D between L_1, L_2 is equal to the distance between any point on L_2 and Π . If we take the point $(4, 6, 7)$ on L_2 then

$$D = \frac{|2(4 - 1) - 11(6 - 3) + (7 - 5)|}{\sqrt{2^2 + 11^2 + 1^2}} = \frac{25}{42}\sqrt{14}$$

3. The normals $\mathbf{n}_1, \mathbf{n}_2$ to the planes are $\mathbf{n}_1 = \langle 1, -1, 1 \rangle$, $\mathbf{n}_2 = \langle 2, 1, 3 \rangle$. The formula for the angle θ between the two planes is $\cos \theta = \frac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = \frac{4}{\sqrt{3}\sqrt{14}} = \frac{2}{21}\sqrt{42}$. Hence $\theta = \cos^{-1} \frac{2}{21}\sqrt{42} = 0.9056 = 51.89^\circ$.
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4. The distance formula gives $D = \frac{|8(3) + (-2)(-5) + (2) - 5|}{\sqrt{8^2 + (-2)^2 + (1)^2}} = \frac{31}{69}\sqrt{69}$.
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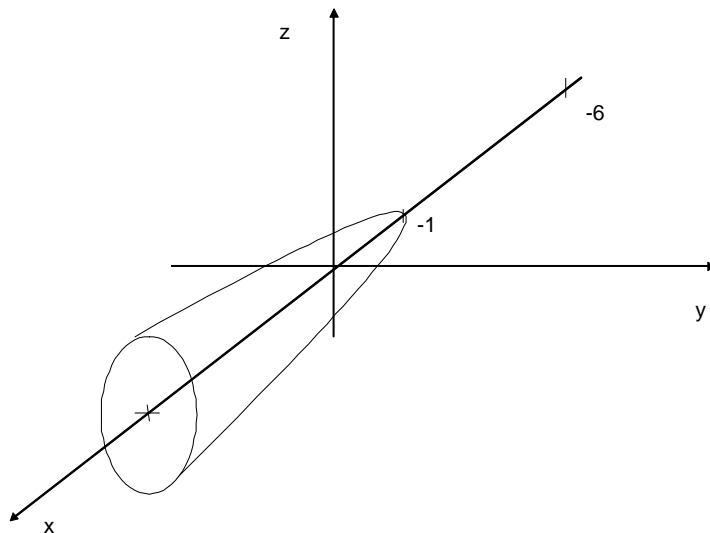
5. We need to find values of t which give points on the line at distance $\sqrt{14}$ from the plane. Substituting the general point on the line in the distance formula and equating to $\sqrt{14}$ gives

$$\sqrt{14} = \frac{|3(2 + 3t) - (1 - t) + 2(8 + 2t) - 4|}{\sqrt{3^2 + (-1)^2 + (2)^2}} = \frac{|17 + 14t|}{\sqrt{14}}$$

Therefore, $17+14t = \pm 14$. Solve to get $t = -\frac{31}{14}, -\frac{3}{14}$. Substitute these values in the equations of the line to obtain the two points $(-\frac{65}{14}, \frac{45}{14}, \frac{25}{7}), (\frac{19}{14}, \frac{17}{14}, \frac{53}{7})$.

6. The normals to the planes are $\mathbf{n}_1 = \langle 1, 1, -1 \rangle$, $\mathbf{n}_2 = \langle 2, 3, -5 \rangle$. The direction \mathbf{v} of the line of intersection is $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 3 & -5 \end{vmatrix} = \langle -2, 3, 1 \rangle$. Therefore, the equations of the line are $x = 2 - 2t$, $y = 3 + 3t$, $z = 1 + t$.
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7. 1. The surface is a *circular paraboloid* with vertex at $(-1, 0, 0)$ and opening in the positive x -direction.
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3. The point $(-6, 0, 0)$ is a point at a distance 5 from the surface.
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8. 1. A sphere centered at the origin of radius 2
2. A sphere centered at the origin of radius 2
3. The upper half of a cone of vertex angle $\frac{\pi}{4}$
4. Changing to cylindrical coordinates, we get $r = 2$, $z \geq 0$. Therefore, we have the upper half of a cylinder with axis along the z -axis of radius 2.
2.