KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 201-05

1. Identify **(2pts)** and sketch **(2pts)** the surface $y = \sqrt{1 + x^2 - z^2}$**.**

 Solution:

$$\begin{aligned} y^2 &= 1 + x^2 - z^2 \\ z^2 - x^2 + y^2 &= 1. \end{aligned}$$

 Therefore, the surface is the right half of a Hyperboid of one sheet.

2. (a) Change **(2pts)** $\left(\sqrt{3}, \frac{\pi}{3}, -1\right)$ From cylindrical to spherical coordinates.

 (b) Change the equation $x = 3$ into Spherical coordinates **(2pts)**.

 Solution:

$$\begin{aligned} \rho &= \sqrt{r^2 + z^2} = 2, \\ \theta &= \frac{\pi}{3}, \\ \cos\varphi &= \frac{z}{\rho} = -\frac{1}{2}, \\ \varphi &= \frac{2\pi}{3}. \end{aligned}$$

 Therefore $\left(\sqrt{3}, \frac{\pi}{3}, -1\right) \to \left(2, \frac{\pi}{3}, \frac{2\pi}{3}\right)$.

 (b)

$$\begin{aligned} x &= \rho \sin\varphi \cos\theta \\ \rho \sin\varphi \cos\theta &= 3 \\ \rho &= 3 \csc\varphi \sec\theta. \end{aligned}$$

3. **(2pts)** Identify and sketch the surface $r^2 = r$.

 Solution:

$$\begin{aligned} r^2 - r &= 0 \Longrightarrow \\ r &= 1 \text{ or } r = 0. \end{aligned}$$

 $r = 1$ is a circular cylinder of raduis 1 with axis along the z-axis. $r = 0$ is the z-axis itslef. Therefore, the surface is the union of the cylinder and the z-axis.