

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
DEPARTMENT OF MATHEMATICS & STATISTICS  
MATH 201-05

1. Identify **(2pts)** and sketch **(2pts)** the surface  $y = \sqrt{1 + x^2 - z^2}$ .

Solution:

$$\begin{aligned}y^2 &= 1 + x^2 - z^2 \\z^2 - x^2 + y^2 &= 1.\end{aligned}$$

Therefore, the surface is the right half of a Hyperboid of one sheet.

2. (a) Change **(2pts)**  $(\sqrt{3}, \frac{\pi}{3}, -1)$  From cylindrical to spherical coordinates.  
(b) Change the equation  $x = 3$  into Spherical coordinates **(2pts)**.

Solution:

$$\begin{aligned}\rho &= \sqrt{r^2 + z^2} = 2, \\ \theta &= \frac{\pi}{3}, \\ \cos \varphi &= \frac{z}{\rho} = -\frac{1}{2}, \\ \varphi &= \frac{2\pi}{3}.\end{aligned}$$

Therefore  $(\sqrt{3}, \frac{\pi}{3}, -1) \rightarrow (2, \frac{\pi}{3}, \frac{2\pi}{3})$ .

(b)

$$\begin{aligned}x &= \rho \sin \varphi \cos \theta \\ \rho \sin \varphi \cos \theta &= 3 \\ \rho &= 3 \csc \varphi \sec \theta.\end{aligned}$$

3. **(2pts)** Identify and sketch the surface  $r^2 = r$ .

Solution:

$$\begin{aligned}r^2 - r &= 0 \implies \\ r &= 1 \text{ or } r = 0.\end{aligned}$$

$r = 1$  is a circular cylinder of radius 1 with axis along the  $z$ -axis.  $r = 0$  is the  $z$ -axis itself. Therefore, the surface is the union of the cylinder and the  $z$ -axis.