KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 201-05
Quiz # 2

1. Find equations of the tangent and normal lines to the graph of the polar curve $r = \sin 2\theta$ at $\theta = \pi/4$.

<u>Solution</u>

At $\theta = \pi/4$, $r = \sin(\pi/2) = 1$, $x = 1/\sqrt{2}$, $y = 1/\sqrt{2}$, $\frac{dr}{d\theta} = 2\cos(\pi/2) = 0$.

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \left.\frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}\right|_{\theta=\pi/4} = -1$$

Slope of the normal $= 1$.

Equation of the tangent

$$y - 1/\sqrt{2} = -\left(x - 1/\sqrt{2}\right).$$

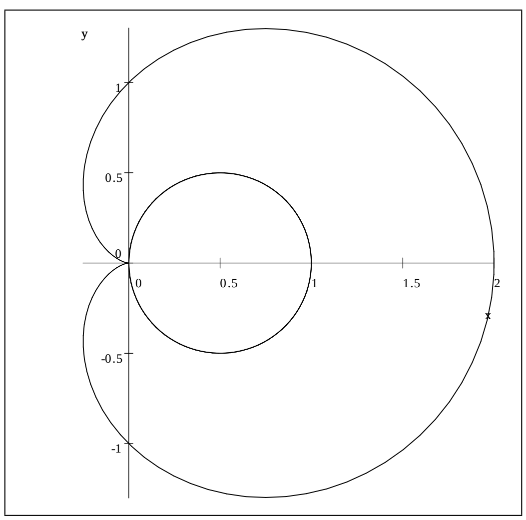
Equation of the normal

$$y - 1/\sqrt{2} = x - 1/\sqrt{2}.$$

2. Find the area inside the curve $r = 1 + \cos\theta$ and outside the curve $r = \cos\theta$.

<u>Solution</u>

The graphs of the curves are shown below.



The required area is

$$\begin{aligned} A &= \int_0^{2\pi} \frac{1}{2}(1+\cos\theta)^2\,d\theta - \int_0^{\pi} \frac{1}{2}(\cos\theta)^2\,d\theta \\ &= \frac{3}{2}\pi - \frac{1}{4}\pi = \frac{5}{4}\pi. \end{aligned}$$

3. The distance from the point $P(x, y, z)$ and the piont $A(1, -2, 0)$ is twice the distance between P and the point $B(0, 1, 1)$. Show that the set of all such points is a sphere and find its center and radius.

Solution

The given relation between the distances gives the equation

$$(x-1)^2 + (y+2)^2 + z^2 = 4\left[x^2 + (y-1)^2 + (z-1)^2\right]$$

Which simplifies to

$$x^2 + y^2 + z^2 + \frac{2}{3}x - 4y - \frac{8}{3}z = -1.$$

Completing the squares, we get

$$\begin{aligned}\left(x+\frac{1}{3}\right)^2 + (y-2)^2 + \left(z-\frac{4}{3}\right)^2 &= -1 + \frac{1}{9} + 4 + \frac{16}{9} \\ &= \frac{44}{9}.\end{aligned}$$

Thus, the center of the sphere is $\left(-\frac{1}{3}, 2, \frac{4}{3}\right)$ and the radius is $\frac{2}{3}\sqrt{11}$.