

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
 DEPARTMENT OF MATHEMATICS & STATISTICS
 MATH 201-04
 Quiz # 1

1. Which points on $x = 3t - t^2, y = t^3 - 12$ have tangents with slope 3?

Answer:

$$\begin{aligned} \frac{dy}{dx} &= \frac{3t^2}{3-2t} = 3 \implies t^2 + 2t - 3 = 0 \\ &\implies t = -3, t = 1 \end{aligned}$$

Inserting the values of t in the equations for x, y we get the two points $(-18, -39)$ and $(2, -11)$.

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(1, 3)$ on the curve $x = e^t, y = 3 \cos t$.

Answer:

$$\begin{aligned} \frac{dy}{dx} &= \frac{-3 \sin t}{e^t}, \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt} \frac{dy}{dx}}{\frac{dx}{dt}} = \frac{e^t (-3 \sin t) + e^t 3 \sin t}{e^{3t}} \\ &= \frac{-3 \cos t + 3 \sin t}{e^{2t}} \end{aligned}$$

At the point $(1, 3), t = 0$ Therefore $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = -3$.

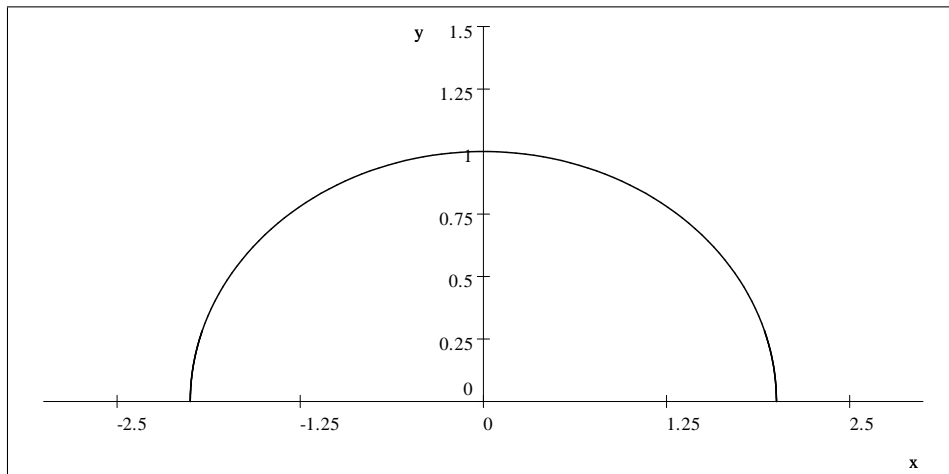
3. Eliminate t and sketch the resulting curve for $x = 2 \sin t, y = \cos t, -\frac{\pi}{2} < t < \frac{\pi}{2}$. Indicate with an arrow the direction in which the curve is traced as t increases.

Answer:

Eliminating t results in the equation

$$\frac{x^2}{4} + y^2 = 1.$$

For the given range of t , we only have the upper half of the ellipse.



The curve is traced clockwise from $x = -2$ to $x = 2$.