KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 201-11
Final Exam
Jan. 22, 2008
Time Allowed: 2 hours

NAME:	ID#:

SHOW ALL YOUR WORK

1. (a) **(3 points)** Set up an integral to compute the area common to both curves $r = \sin 2\theta$ and $r = \cos 2\theta$. (Do not perform the integration.)

   (b) **(2 points)** Find all points of intersection of the curves $r = \cos\theta$ and $r = 1 - \cos\theta$.

2. (a) **(2 points)** Identify and make a clear sketch of the surface $x^2 - y^2 + z^2 - 2x + 2y - 4z + 4 = 0$.

   (b) **(3 points)** Make a clear sketch of the solid described by the inequalities $\varphi \leq \frac{\pi}{2},\ \rho \leq 2$ and find its projection in the xy-plane.

3. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

where T is measured in °C and x, y, z in meters.

(a) **(2 points)** Find the rate of change of the temperature at the point $P(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.

(b) **(2 points)** In which direction does the temperature increase fastest at P?

(c) **(1 points)** Find the maximum rate of increase at P.

4. **(5 points)** Find the maximum volume of the box with 3 of its edges along the positive coordinate axis and one vertex in the plane $x + 2y + 3z = 9$.

5. Evaluate the integrals

(a) (**2 points**) $\int_0^1 \int_{x^2}^1 x^3 \sin\left(y^3\right) dydx,$ (**3 points**) $\int_0^a \int_0^{\sqrt{a^2-y^2}} \left(x^2+y^2\right)^{3/2} dxdy.$

6. **(5 points)** Rewrite the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f\left(x, y, z\right) dzdydx$$

as an equivalent iterated integral in the five other orders.

7. Change the integral

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}dzdxdy$$

into an equivalent integral (a) **(2 points)** in spherical coordinated and (b) **(3 points)** in Cylindrical coordinates.