1. (a) Identify the curve $r = \sec\theta\tan\theta$ by changing to cartisian coordinates.

Solution:

$$\begin{aligned} r &= \sec\theta\tan\theta \\ r\cos\theta &= \tan\theta \\ x &= \frac{y}{x} \\ y &= x^2 \end{aligned}$$

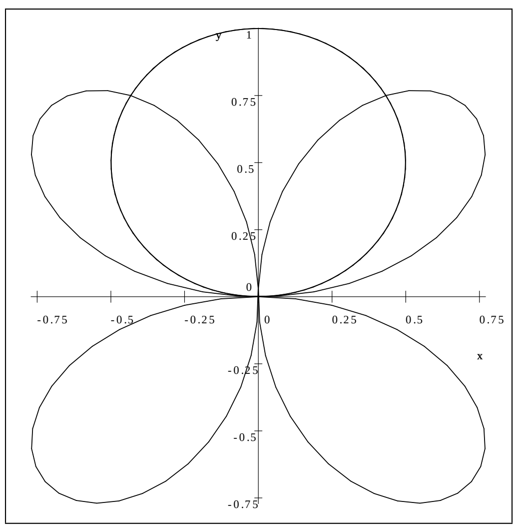
The curve is a parabola.

(b) Find a polar equation, in simplified form for the curve represented by the rectangular equatoin $x^2 - y^2 = 1$.

Solution:

$$\begin{aligned} r^2\cos^2\theta - r^2\sin^2\theta &= 1 \\ r^2\left(\cos^2\theta - \sin^2\theta\right) &= 1 \\ r^2\cos 2\theta &= 1 \\ r^2 &= \csc 2\theta. \end{aligned}$$

2. Set up an integral to compute the area inside both of the curves $r = \sin 2\theta$



and $r = \sin\theta$.

Solution:

To get the points of intersection:

$$\begin{aligned} \sin 2\theta &= \sin\theta \\ 2\sin\theta\cos\theta &= \sin\theta \\ \sin\theta &= 0 \text{ or } \cos\theta = \frac{1}{2} \\ \theta &= 0 \text{ or } \theta = \frac{\pi}{3}. \end{aligned}$$

The area is given by

$$2\left[\frac{1}{2}\int_0^{\pi/3}\sin^2\theta d\theta+\frac{1}{2}\int_{\pi/3}^{\pi/2}\sin^2\theta d\theta\right].$$

3. Find a vector that is orthogonal to the vector between the two points $P\,(1,-1)\,,\ Q\,(2,3)$ and has length 3.

   Solution:

   The vector $\overrightarrow{PQ}=\langle 2-1,3+1\rangle=\langle 1,4\rangle$. An orthogonal vector is $\langle -4,1\rangle$. A unit orthogonal vector is $\frac{1}{\sqrt{17}}\langle 1,4\rangle$. An orthogonal vector of length 3 is $\frac{3}{\sqrt{17}}\langle 1,4\rangle=\left\langle \frac{3}{\sqrt{17}},\frac{12}{\sqrt{17}}\right\rangle$.