KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
MATH 201-11
Quiz # 5

1. Use Lagrange multipliers to find the maximum and minimum of the function $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 1$.

Solution
Applying the Lagrange Multiplier mathod we get the system of equations

$$\begin{aligned} x &= \lambda xy, \\ x^2 &= 2\lambda y, \\ x^2 + y^2 &= 1. \end{aligned}$$

The first equation gives $x = 0$ or $y = \lambda$.

If $x = 0$, the third equation gives $y = \pm 1$ and therefore, we have the two critical pionts $(0, \pm 1)$.

If $y = \lambda$ the second equation becomes $x^2 = 2y^2$. This together with the thrid equation give $3y^2 = 1$. Then $y = \pm \frac{1}{\sqrt{3}}$, from which we get $x = \pm\sqrt{\frac{2}{3}}$. Therefore, we have the four critical points $\left(\pm\sqrt{\frac{2}{3}}, \pm\frac{1}{\sqrt{3}}\right)$. To find the max and min of $f$ we substitute the critical points in the expression of $f$.

$$\begin{aligned} f(0, \pm 1) &= 0, \\ f\left(\pm\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right) &= \frac{2}{3\sqrt{3}}, \\ f\left(\pm\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right) &= -\frac{2}{3\sqrt{3}}. \end{aligned}$$

Therefore, $f_{\max} = \frac{2}{3\sqrt{3}}$, occuring at the points $\left(\pm\sqrt{\frac{2}{3}}, \frac{1}{\sqrt{3}}\right)$ and $f_{\min} = -\frac{2}{3\sqrt{3}}$ occuring at the points $\left(\pm\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{3}}\right)$.