1. (a) Find the parametic equations of the straight line L through the point $P\,(1,-2,2)$ that is perpindicular to the plane $\Pi:\ x-2y+z=8.D$

Solution:

Direction of the line L is $\langle 1,-2,1\rangle$ (the same as the normal to the plane Π).

Equations of L are:

$$\begin{aligned} x &= 1+t \\ y &= -2-2t \\ z &= 2+t. \end{aligned}$$

(b) Where dose the line L intersect the plane Π?

Solution:

Pet $x=1+t,\ y=-2-2t,\ z=2+t$ in the equation of Π. This gives

$$(1+t)-2\,(-2-2t)+(2+t)=8$$

Solve for t to get $t=\frac{1}{6}$. Calculate x,y,z corresponding to $t=\frac{1}{6}$ to obtain the point of intersection

$$\left(\frac{7}{6},-\frac{7}{3},\frac{13}{6}\right).$$

(c) Find the distance between the point P and the plane Π in two different ways.

Solution:

Using the distance formula,

$$d=\frac{|1+4+2-8|}{\sqrt{1+4+1}}=\frac{1}{\sqrt{6}}.$$

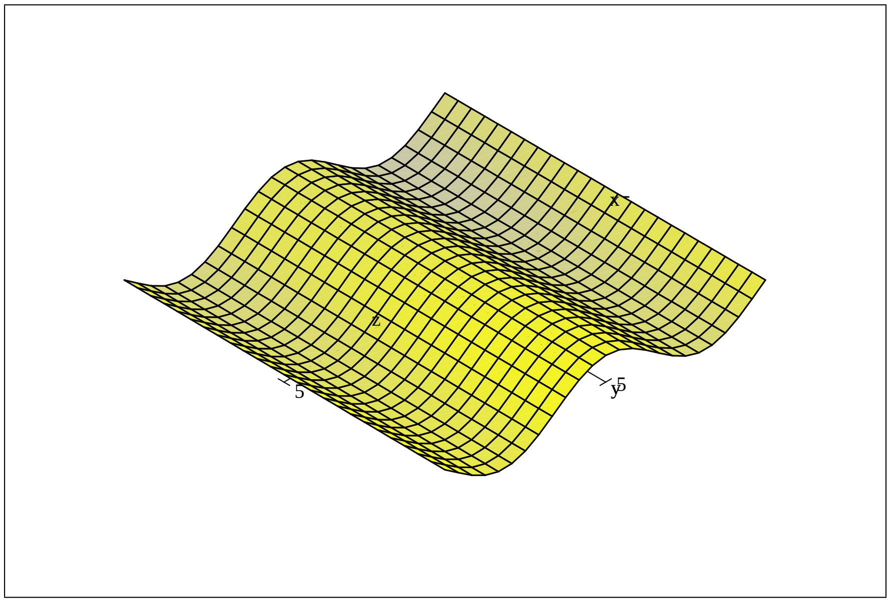
The distance is also the distance between the two points $(1,-2,2)$ and $\left(\frac{7}{6},-\frac{7}{3},\frac{13}{6}\right)$. Therefore,

$$d=\sqrt{\left(1-\frac{7}{6}\right)^2+\left(-2+\frac{7}{3}\right)^2+\left(2-\frac{13}{6}\right)^2}=\frac{1}{\sqrt{6}}.$$

2. Describe and sketch the surface $z=\cos x$.

Solution:

The surface is a cylinder with directrix along the Y-axis. The graph is showm below.


x
z
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y
5