

1. **(3points)** Find the length of the curve  $x = 1 + 3t^2, y = 4 + 2t^3, 0 \leq t \leq 1$ .

$$x' = 6t, \quad y' = 6t^2.$$

$$L = \int_0^1 \sqrt{36t^2 + 36t^4} dt = 6 \int_0^1 t\sqrt{1+t^2} dt$$

put  $u = 1 + t^2$ , and  $du = 2t dt$ . Then

$$L = \frac{1}{2} \int_1^2 \sqrt{u} du = \frac{2}{3} \sqrt{2} - \frac{1}{3}.$$

2. **(4points)** Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for the parametric curve  $x = t + \ln t, y = t - \ln t$ .

$$dy/dx = \frac{dy/dt}{dx/dt} = \frac{1 - \frac{1}{t}}{1 + \frac{1}{t}} = \frac{t-1}{t+1}.$$

$$d^2y/dx^2 = \frac{(1 + \frac{1}{t})\frac{1}{t^2} + \frac{1}{t^2}(1 - \frac{1}{t})}{(1 + \frac{1}{t})^3} = \frac{2t}{(t+1)^3}.$$

3. **(3points)** Eliminate  $t$  to find the Cartesian equation of the curve  $x = t^2 - 1, y = t^2 - 2t + 3, -1 \leq t \leq 0$ .

From the equation for  $x, t = -\sqrt{x+1}$  (observe that  $t$  is negative). Substituting into the equation for  $y$  we get

$$y = x + 1 + 2\sqrt{x+1} + 3.$$