1. Give the missing values:

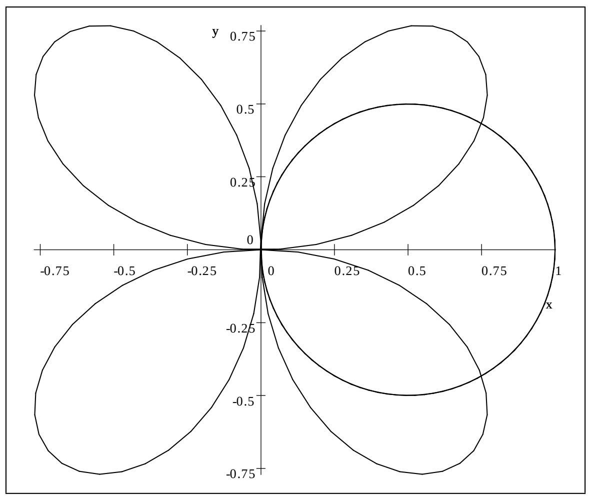
   (a) $\left(-2, -\frac{\pi}{2}\right)$ in polar coordinates $= (0\ \ ,\ 2\ \ )$ in rectangular coordinates.

   (b) $\left(2, \frac{5\pi}{4}\right)$ in polar coordinates $= \left(-2,\ \frac{\pi}{4}\ \ \right)$ in polar coordinates.

   (c) $\left(3, \frac{7\pi}{6}\right)$ in polar coordinates $= \left(\ \ -3\ \ , \frac{\pi}{6}\right)$ in polar coordinates.

   (d) $(2, -2)$ in rectangular coordinates $= \left(\ \ -2\sqrt{2}\ \ , \frac{3\pi}{4}\right)$ in polar coordinates.

2. **(4 points)** Set up an integral to compute the common area between the rose $r = \sin 2\theta$ and the circle $r = \cos\theta$.



Points of intersection:

$$\begin{aligned}
\sin 2\theta &= \cos\theta \\
2\sin\theta\cos\theta &= \cos\theta \\
\cos\theta &= 0 \text{ or } \sin\theta = \frac{1}{2} \\
\theta &= -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}
\end{aligned}$$

The area is given by

$$A = 2\int_0^{\pi/6} \frac{1}{2}\sin^2 2\theta d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}\cos^2 2\theta d\theta.$$

3. A parametric curve is said to cross itself if it passes through the same point (x, y) for two distinct values of the parameter t.

   (a) Show that the curve $x = t^3 - 4t,\ y = t^2$ crosses itself at the point $(0, 4)$ and give the values of t at which the curve crosses itself.

(b) Find the equations of the two tangent lines to the curve in part (a) at the point $(0, 4)$.

(a) $x = 0$ when $t = -2, 0, 2$ and $y = 4$ when $t = -2, 2$. Thus the curve passes through the point $(0, 4)$ when $t = -2, 2$.

(b)

$$\frac{dy}{dx} = \frac{2t}{3t^2 - 4}.$$

At $t = 2$, $\frac{dy}{dx} = \frac{1}{2}$ and at $t = -2$, $\frac{dy}{dx} = -\frac{1}{2}$. Therefore, the equations of the tangents are

$$y - 4 = \frac{1}{2}x$$

and

$$y - 4 = -\frac{1}{2}x.$$

4. (a) Show that the equation of the cardioid $r = 1+\cos\theta$ can be written as $r = 2\cos^2\frac{\theta}{2}$.

(b) Find the arclenght of the cardioid in part (a).

(a) The half angle identitiy states that

$$2\cos^2\frac{\theta}{2} = (1 + \cos\theta).$$

Therefore,

$$r = 1 + \cos\theta = 2\cos^2\frac{\theta}{2}.$$

(b) The length of the cardioid is given by

$$\begin{aligned}
L &= \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \\
&= \int_0^{2\pi} \sqrt{(1 + \cos\theta)^2 + \sin^2\theta d\theta} \\
&= \int_0^{2\pi} \sqrt{2 + 2\cos\theta} d\theta \\
&= \sqrt{2}\int_0^{2\pi} \sqrt{1 + \cos\theta} d\theta \\
&= \sqrt{2}\int_0^{2\pi} \sqrt{2\cos^2\frac{\theta}{2}} d\theta \\
&= 2\int_0^{2\pi} \left|\cos\frac{\theta}{2}\right| d\theta \\
&= 2\left(\int_0^{\pi/2} \cos\frac{\theta}{2} d\theta - \int_{\pi/2}^{3\pi/2} \cos\frac{\theta}{2} d\theta + \int_{3\pi/2}^{2\pi} \cos\frac{\theta}{2} d\theta\right) \\
&= 8.
\end{aligned}$$

5. (a) Use triple scalar product to determine whether the points $P\left((1, 0, 1)\right)$, $Q\left(2, 4, 6\right)$, $R\left(3, -1, 2\right)$ and $S\left(6, 2, 8\right)$ lie in the same plane.

(b) Find the scalar and vector projections of the vector $\mathbf{u} = \langle 2, -3, 1 \rangle$ onto the vector $\mathbf{v} = \langle 1, 6, -2 \rangle$.

(a) The 4 points will be coplanar if the parallelipipped with sides PQ, PR, PS has volume zero. We use triple scalar product to compute that volume:

$$\begin{aligned} \overrightarrow{PQ} &= \langle 1, 4, 5 \rangle, \\ \overrightarrow{PR} &= \langle 2, -1, 1 \rangle, \\ \overrightarrow{PS} &= \langle 5, 2, 7 \rangle. \\ \overrightarrow{PQ} \cdot \left(\overrightarrow{PR} \times \overrightarrow{PS} \right) &= \begin{vmatrix} 1 & 4 & 5 \\ 2 & -1 & 1 \\ 5 & 2 & 7 \end{vmatrix} = 0. \end{aligned}$$

Therefore, the 4 points lie in the same plane.

(b)

$$\begin{aligned} \text{Comp}_{\mathbf{v}}\mathbf{u} &= \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|} = -\frac{18}{\sqrt{41}}. \\ \text{Proj}_{\mathbf{v}}\mathbf{u} &= \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} = -\frac{18}{\sqrt{41}} \langle 1, 6, -2 \rangle \\ &= \left\langle -\frac{18}{\sqrt{41}}, -\frac{108}{\sqrt{41}}, \frac{36}{\sqrt{41}} \right\rangle. \end{aligned}$$